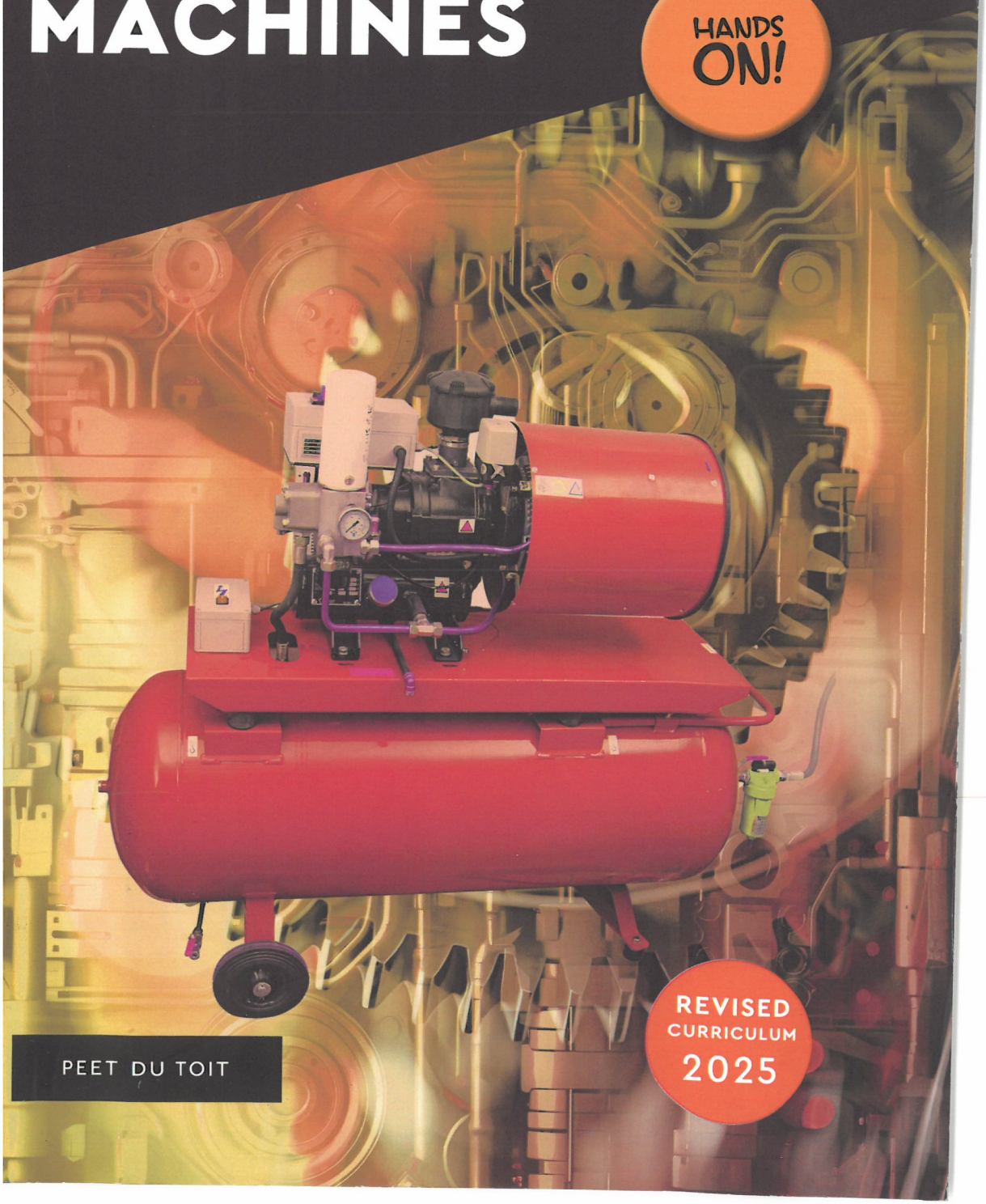
N6

POWER MACHINES

HANDS ON!



REVISED CURRICULUM 2025

PEET DU TOIT

N6

Power Machines

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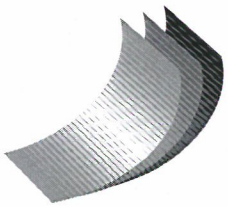

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MODULE

1 *Internal combustion engines*

By the end of this module, students should be able to:

- illustrate an understanding of air standard engines through a pressure-versus-volume diagrams, and number the principal points according to the compression, combustion, expansion and exhaust strokes for the:
  - Otto cycle
  - diesel cycle
  - dual cycle;
- consider actual and adiabatic assumptions for Joule cycle;
- perform calculations in relation to air standard engines using the relevant formulae with regard to:
  - volumes
  - brake and indicated power
  - efficiencies
  - specific fuel consumption
  - relative efficiency ratio; and
- perform calculations based on the energy balance and/or draw up a detailed energy balance table and convert units to the specified units of heat per unit of time with respect to:
  - fuel power
  - brake power
  - friction power
  - cooling water
  - exhaust gases
  - radiation (remainder).

Introduction

An internal combustion engine (ICE) is a heat engine where fuel combustion occurs internally. It operates on the principle of converting the chemical energy of fuel into mechanical energy through the process of combustion, which occurs within a confined space known as the combustion chamber.

This process involves igniting a fuel-air mixture, resulting in the rapid expansion of gases that exert force on engine components, typically pistons. These pistons then convert linear motion into rotational motion via a crankshaft, ultimately powering vehicles and machinery.

Internal combustion engines can be classified into various types, including gasoline and diesel engines, which differ primarily in their ignition methods and fuel types.

Since their development in the 19th century, ICEs have become the dominant power source for automobiles, aircraft, and many other applications, significantly impacting modern society and the economy.

1.1 Operations performed in an internal combustion engine

In an internal combustion engine, particularly in a four-stroke engine, four distinct operations occur in a cyclical manner to convert fuel into mechanical energy.

These operations are:

Intake stroke

The cycle begins with the piston moving downwards, creating a vacuum that opens the intake valve. This allows an air-fuel mixture (petrol engines) or just air (diesel engines) to be drawn into the combustion chamber until the piston reaches the bottom of its stroke. This position is referred to as the bottom dead centre (BDC).

Compression stroke

Once the piston moves back up, both the intake and exhaust valves close, sealing the mass in the cylinder. The piston compresses this mass, increasing its pressure and temperature in preparation for ignition.

Power stroke

Just before the end of the compression stroke, a spark plug ignites the compressed air-fuel mixture in a petrol engine. The rapid expansion of gases from the combustion forces the piston down, generating power that turns the crankshaft. In a diesel cycle there is no spark plug and fuel is injected into the cylinder , causing combustion to take place spontaneously due to its high compression ratio. This usually takes place just after the piston has reached the top of its stroke. This position is referred to as the top dead centre (TDC).

Exhaust stroke

After the power stroke, the piston moves back up again, this time with the exhaust valve open. This motion expels the spent combustion gases from the cylinder, completing the cycle and allowing the process to begin again.

The enlarging of the volume of a system is called expansion, while the reduction in the volume of a system is called compression. The gas expands when the pressure on the gas is removed and compresses when the pressure is imposed on the gas. During these processes, heat could be added or removed from the system, depending on the type of process taking place.

Basic concepts

Every science has its own unique vocabulary associated with it. Precise definition of basic concepts forms a sound foundation for development of a science and prevents possible misunderstandings. Careful study of these basic concepts is essential for a good understanding of topics in thermodynamics.

Closed system

It consists of a fixed amount of mass and no mass can cross its boundary. Energy in the form of heat or work can cross the boundary and the volume of a closed system does not have to be fixed.

Process

Any change a system undergoes from one equilibrium state to another is called a process. It describes the path followed when the properties like volume, pressure and temperature changes from an initial state to a final state.

Absolute temperature

It is measured in Kelvin and not degrees Celsius. If the temperature of a gas is given in degrees Celsius, use the following conversion:

$$K = °C + 273$$

Standard temperature and pressure (STP)

Refers to a condition where the absolute pressure is atmospheric pressure at sea level (P=101,325 kPa) and absolute temperature is 0°C (T=273 K).

Normal temperature and pressure (NTP)

Refers to a condition where the absolute pressure is atmospheric pressure at sea level (P=101,325 kPa) and absolute temperature is 15°C (T=288 K).

Specific heat capacity at constant volume (C_v)
It is the amount of heat required to raise or lower the temperature of a unit mass by one degree at constant volume (J/kg.K).

Specific heat capacity at constant pressure (C_p)
It is the amount of heat required to raise or lower the temperature of a unit mass by one degree at constant pressure (J/kg.K).

Specific heat ratio (γ)
The specific heat ratio is used in calculations during a process where no heat transfer takes place. It is defined as the ratio of the specific heat of the gas at constant pressure to its specific heat at constant volume and has no unit.

$$\gamma = \frac{C_p}{C_v}$$

Characteristic gas constant (R)
It is a constant used in the characteristic gas equation which is used to do calculations on the behavior of different gasses under certain conditions (J/kg.K).

$$R = C_p - C_v$$

Internal energy (U)
Internal energy is the sum of the kinetic energies (due to its motion) and the potential energies of the molecules (due to its attraction between molecules). If the temperature in a system increases, the change in internal energy will be positive. If the temperature in a system decreases, the change in internal energy will be negative.

$$\Delta U = mC_v(T_2 - T_1)$$

Work done (W)
Work is done when a force acting on a system causes a change in volume to take place. When the volume of a system is increased, the work done is positive and is seen as work done by the gas. When the volume of a system is decreased, the work done is negative and is seen as work done on the gas. The work done is determined by calculating the area under the PV-diagram.

Heat energy (Q)
Heat is energy transferred between systems due to a temperature difference between them. If heat is added to a system, it is positive. If heat is rejected from the system, it is negative.

Conservation of energy equation

The conservation of energy equation ties in with the first law of thermodynamics which states that energy cannot be destroyed, it can only be converted to another form of energy.

$$Q = \Delta U + W$$

The general gas law

It is used to determine the relations between volume, pressure, temperature and mass in a system for any process.

$$\frac{P_1 V_1}{T_1} = mR = \frac{P_2 V_2}{T_2}$$

The five gas processes are as follows:

- Isochoric: Constant volume process
- Isobaric: Constant pressure process
- Isothermal: Constant pressure process
- Adiabatic: Pressure, volume and temperature change, but no heat flow takes place
- Polytropic: Pressure, volume and temperature change, while heat flow takes place

Process	**Isochoric**	**Isobaric**	**Isothermal**	**Adiabatic**	**Polytropic**
Diagram					
Law	$V_1 = V_2$	$P_1 = P_2$	$PV = C$ $T_1 = T_2$	$PV^{\gamma} = C$ $Q = 0$	$PV^{n} = C$ $Q \neq 0$
Ratios	$\frac{P_1}{P_2} = \frac{T_1}{T_2}$	$\frac{V_1}{V_2} = \frac{T_1}{T_2}$	$\frac{P_1}{P_2} = \frac{V_2}{V_1}$	$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^{\gamma}$ $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$ $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$	$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^{n}$ $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$ $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1}$
ΔU	$mC_v(T_2 - T_1)$	$mC_v(T_2 - T_1)$	0	$mC_v(T_2 - T_1)$	$mC_v(T_2 - T_1)$
W	0	$P(V_2 - V_1)$	$PV\ln\left(\frac{V_2}{V_1}\right)$	$\frac{P_1V_1 - P_2V_2}{\gamma - 1}$	$\frac{P_1V_1 - P_2V_2}{n - 1}$
Q	$mC_v(T_2 - T_1)$	$mC_p(T_2 - T_1)$	$PV\ln\left(\frac{V_2}{V_1}\right)$	0	$W + \Delta U$

Table 1.1: The five gas processes

Terminology used for internal combustion engines

To make it easier to understand all the calculations that are required when dealing with IC engines, here is a list of all the different volumes, powers and efficiencies that will be encountered.

Volumes

- **Cylinder volume (V_1)**
  This is the total volume of the cylinder when it is at bottom dead centre.
- **Clearance volume (V_2)**
  This is the minimum volume in the cylinder when it is at top dead centre.
- **Stroke volume (V_s)**
  This is the difference between cylinder volume and clearance volume. It depends on the diameter and stroke length of the piston.

$$V_s = LA = V_1 - V_2$$

Where
L is stroke length (m)
A is cross-sectional area (m^2)

Compression ratio (r_c): It is the ratio between the cylinder volume and the clearance volume.

$$r_c = \frac{V_{cyl}}{V_c} = \frac{V_1}{V_2}$$

Powers
Fuel power: The power supplied by the fuel (input power)

$$P_f = \dot{m}_f \times CV$$

Where
$\dot{m}_f$ is mass rate of fuel (kg/s)
CV is calorific value of fuel (J/kg)

Indicated power: The power developed in the cylinder

$$P_i = p_{im} \times LANEx$$

Where
p_{im} is indicated mean effective pressure in cylinder (Pa)
L is stroke length of piston (m)
A is cross-sectional area of cylinder (m^2)

N is rotational frequency of crank (rps)
E is 1 for two stroke and 0,5 for four stroke engines
x is number of cylinders

Brake power: The power available on the crankshaft (output power)

$$P_b = p_{bm} \times LANEx$$

$$P_b = 2\pi NT$$

Where
p_{bm} is brake mean effective pressure in cylinder (Pa)
T is brake torque on crankshaft (Nm)

Friction power: The power lost due to friction in cylinder

$$P_\mu = P_i - P_b$$

Efficiencies

Let's look at efficiencies in the cycle.

Air standard efficiency

It represents the efficiency of an ideal thermodynamic cycle, where air is assumed to be the working fluid. This efficiency serves as a benchmark for evaluating the performance of real engines by comparing their actual efficiencies to the theoretical maximum efficiency achievable under ideal conditions

$$\eta_{as} = \frac{W}{Q_{in}}$$

or

$$\eta_{as} = 1 - \frac{Q_{out}}{Q_{in}}$$

Where
W is work done of the cycle (J)
Q_{in} is heat supplied (J)
Q_{out} is heat rejected (J)

Indicated thermal efficiency: It compares the indicated power in the cylinder to the fuel power (cylinder to fuel)

$$\eta_{it} = \frac{P_i}{P_f}$$

Brake thermal efficiency: It compares the brake power on the crankshaft to the fuel power (crank to fuel)

$$\eta_{bt} = \frac{P_b}{P_f}$$

Mechanical efficiency: It compares the brake power to the indicated power (crank to cylinder)

$$\eta_m = \frac{P_b}{P_i}$$

Indicated efficiency ratio: It compares the actual indicated efficiency to the ideal air standard efficiency

$$\eta_{ir} = \frac{\eta_{it}}{\eta_{as}}$$

Brake efficiency ratio: It compares the actual brake efficiency to the ideal air standard efficiency

$$\eta_{br} = \frac{\eta_{bt}}{\eta_{as}}$$

Indicated mean effective pressure: It is the average pressure during the complete cycle process

$$p_{im} = \frac{W}{V_s}$$

$$BSFC = \frac{m_f/h}{P_b}$$

$$ISFC = \frac{m_f/h}{P_i}$$

1.1.1 Air standard engine cycles

Air standard cycle engines are a simplified way to study how internal combustion engines work. They use the idea that air, treated as an ideal gas, moves through a closed loop, which makes it easier to analyze the engine's performance.

In this model, it is assumed that certain processes happen in an ideal manner, such as compressing the air without any heat loss (isentropic compression) and adding heat at a constant volume.

These assumptions help designers understand different types of engines, like Otto and Diesel engines, by breaking down their operations into clear steps. By using these simplified models, engineers can evaluate how well an engine performs and find ways to improve its efficiency.

This approach is essential for designing better engines that meet modern environmental and energy standards.

The constant volume or Otto cycle

In petrol and gas engines, the heat is taken in rapidly from the combustion of the fuel, while there is negligible movement of the piston at the top-dead-centre (TDC) position. Similarly, the heat is rejected rapidly from the cylinder when the exhaust valve opens at or about bottom-dead-centre (BDC). So, the heat exchange operations occur while the volume of the gases in the cylinder remains constant. The ideal cycle corresponding to this arrangement is called the **constant volume cycle**.

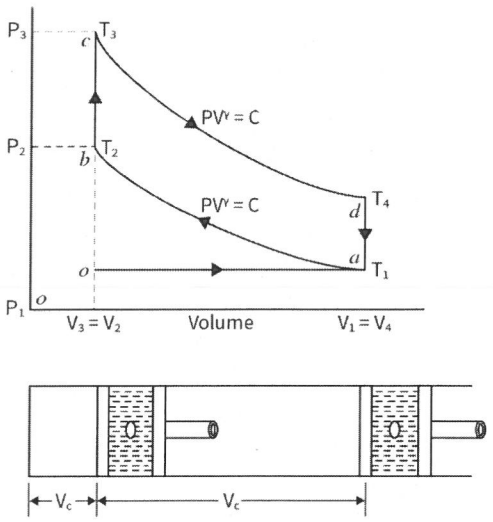


Figure 1.1: Otto cycle

For the purposes of this book, the working agent will be assumed to be air in a cylinder that receives heat perfectly when the piston is at inner dead centre, and rejects heat perfectly when the position is at outer dead centre.

The cycle commences at *a*, where the charge of air is compressed adiabatically to *b*, so that no heat flow takes place while the pressure increases from P_1 to P_2, the temperature increases from T_1 to T_2 and the volume decreases from V_1 to V_2, the clearance volume (V_c).

From *b* to *c*, the heat is added at constant volume, while the pressure and temperature increase from P_2 and T_2 to P_3 and T_3 respectively.

From *c* to *d*, the air expands adiabatically from volume V_3 to V_4 and does work on the piston. The pressure and temperature fall from P_3 and T_3 to P_4 and T_4 respectively, and no heat is taken in or given out during the process. From *d* to *a*, heat is rejected at constant volume (exhaust stroke) as the pressure and temperature fall to the original values of P_1 and T_1.

The ideal or **air standard efficiency** η_a can now be obtained for an engine working on such a cycle.

$$\eta_{as} = \frac{((\text{Heat taken in cycle}) - (\text{Heat rejected cycle}))}{\text{Heat taken in cycle}}$$

$$\eta_{as} = 1 - \frac{\text{Heat rejected per cycle}}{\text{Heat taken in per cycle}}$$

The expansion and compression operations are both adiabatic, where there is no heat flow across the boundaries. Heat required to raise m.kg of gas from T_1 to T_2:

$m \times C_v(T_2 - T_1)$, where C_v = specific heat capacity at constant volume.
The result is shown below.
Heat added = $m \times C_v(T_3 - T_2)$
Heat rejected = $m \times C_v(T_4 - T_1)$

$$\eta_{as} = 1 - \frac{m \times C_V(T_4 - T_1)}{m \times C_V(T_3 - T_2)}$$
$$= \frac{T_4 - T_1}{T_3 - T_2} \quad \ldots\ldots\ldots\

$$\eta_{as} = 1 - \frac{\frac{T_1 T_3}{T_2} - T_1}{T_3 - T_2}$$

$$= 1 - \frac{\left(T_1 \frac{(T_3 - 1)}{T_2}\right)}{T_3 - T_2}$$

$$= \frac{\frac{T_1}{T_2}(T_3 - T_2)}{T_3 - T_2}$$

$$= 1 - \frac{T_1}{T_2} \qquad \text{but } \frac{T_1}{T_2} = \left(\frac{1}{r}\right)^{\gamma - 1}$$

$$= 1 - \left(\frac{1}{r}\right)^{\gamma - 1} \qquad \text{but } r = \text{expansion or compression ratio}$$

Note
ASE or η_{as} is usually a percentage.

The constant volume cycle in practice

The shape of the actual indicator diagram traced out by an engine follows the ideal diagram fairly closely cycle.

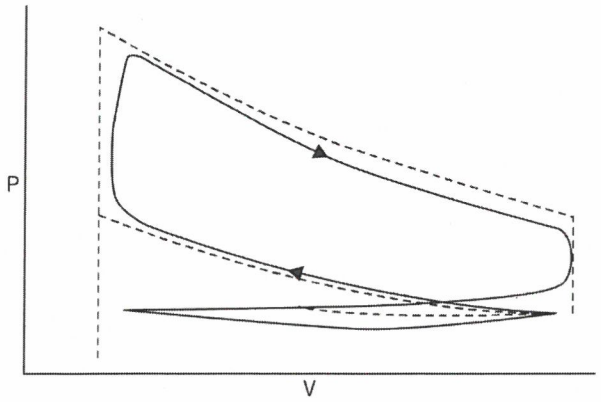


Figure 1.2

However, the action of a practical engine differs in many respects from the ideal cycle for several reasons.

- The heat is released from within the working agent, which is a combustible mixture and not air alone.
- Complicated exchanges of heat take place between the cylinder walls and the cooling jackets throughout the cycle.
- The heat from the charge takes an appreciable time to be released, so that the heat intake is not exactly at constant volume.

Similarly, the exhaust valve takes some time to open as shown by the rounding-off of the diagram at this point, so that heat rejection also does not take place at exactly constant volume.

Nevertheless, the expression for ASE gives a useful point of comparison for all engines working on this cycle. The first important point is that thermal efficiency increases as the compression ratio increases. This fact is proved in practice.

In the design of internal combustion engines, every effort is made to use as high a compression ratio as possible. Unfortunately, with petrol and gas engines, where a mixture of fuel and air is compressed into the clearance space during the compression stroke, the maximum compression ratio that can be used is about 8,7 : 1, due to an occurrence called detonation or pinking.

Example 1.1

Calculate the air standard efficiency for an engine working on the constant volume cycle with a compression ratio of 6,5: 1. Assume gamma = 1,4.

Solutions

$$\eta_a = 1 - \left(\frac{1}{r}\right)^{\gamma-1} = 1 - \left(\frac{1}{6,5}\right)^{1,4-1} = 0,528 = 52,8\%$$

$$=$$

$$= 1 - 0$$

Example 1.2

The η_a of an Otto cycle engine is 56,5%. The bore and stroke of the engine are equal. The clearance volume is 9,84 cm^3. Assume gamma to be 1,4.

1. Calculate the volume–compression ratio (r).
2. Calculate the swept volume of the engine (in cubic centimetres).
3. Calculate the diameter (D) of the engine (in millimetres).

Solutions

1. Find the volume–compression ratio (r).

$$\eta_{as} = 1 - \left(\frac{1}{r}\right)^{\gamma-1}$$

$$0,565 = 1 - \left(\frac{1}{r}\right)^{1,4-1}$$

$$\therefore r = 8$$

$$\therefore \text{compression ratio} = 8:1$$

2. Find the swept volume of the engine.

$\frac{V_1}{V_2} = r$

$= 8$

and $V_2 = 9{,}84\ cm^3$

Compression ratio $= 8:1$

$\therefore$ swept volume $= 7 \times V_2$

$= 7 \times 9{,}84$

$= 68{,}88\ cm^3$

3. Swept volume = area of piston × stroke

$= \frac{\pi}{4} D^2 \times D$

$68{,}88 = \frac{\pi}{4} D^3$

$D = \sqrt[3]{\frac{68{,}88 \times 4}{\pi}}$

$= \sqrt[3]{87{,}5}$

$= 4{,}44\ cm$

$= 44{,}4\ mm$

Example 1.3

In the ideal constant volume cycle, the temperatures at the beginning and the end of compression are 55 °C and 350 °C respectively. Gamma = 1,4.

1. Calculate the volume compression ratio.
2. Calculate the air standard efficiency

Solutions

1.

$$r_v = \frac{V_1}{V_2} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma - 1}}$$

$$= \left(\frac{623}{328}\right)^{\frac{1}{1{,}4-1}} = 4{,}972$$

2.

$$\eta_{as} = 1 - \left(\frac{1}{r}\right)^{\gamma - 1}$$

$$= 1 - \left(\frac{1}{4{,}972}\right)^{1{,}4-1}$$

$$= 0{,}474 = 47{,}352\%$$

Example 1.4

The cylinder of an engine have a swept volume of 1 639 cm^3 and a clearance volume of 327,7 cm^3. It operates on the constant volume (Otto) cycle where C_p = 1 kJ/kg.K and C_v = 0,71 kJ/kg.K.

Calculate the value of the air standard efficiency.

Solutions

$V_1 = V_s + V_c$

$= 1\,639 + 327{,}7$

$= 1\,966{,}7\ cm^3$

$r = \frac{V_1}{V_2} = \frac{1\,996{,}7}{327{,}7} = 6$

$\gamma = \frac{C_P}{C_V} = \frac{1}{0{,}71} = 1{,}41$

$\eta_{as} = 1 - \left(\frac{1}{r}\right)^{\gamma-1}$

$= 1 - \left(\frac{1}{6}\right)^{0{,}41} = 52\%$

Example 1.5

A six-cylinder, four-stroke petrol engine was tested at 3 000 r/min. The following data was recorded: duration of test = 12 min.; petrol used = 5,4 kg; heat value of fuel used = 40 MJ/kg; cylinder diameter = 100 mm; stroke length = 115 mm; clearance volume = 13% of stroke volume; effective length of brake arm = 2,5 m; mass on brake arm = 11,5 kg; indicated mean effective pressure = 800 kPa. Assume gamma for air = 1,4;

1. Calculate the brake power output.
2. Calculate the indicated power output.
3. Calculate the brake thermal (overall) efficiency.
4. Calculate the air standard efficiency.
5. Calculate the efficiency ratio based on brake power output.

Solutions

1. $P_b = 2\pi NT$

   $= 2 \times \pi \times 50 \times 2{,}5 \times 11{,}5 \times 10\ Nm$

   $= 90{,}32\ kW$

$N = 3\,000\ r/min.;\ T = 2{,}5 \times 11{,}5 \times 10\ Nm$

2. $P_i = \frac{pLANEx}{60}$

$= 800 \times 0{,}115 \times \frac{\pi}{4} \times 0{,}1^2 \times 50 \times 0{,}5 \times 6$

$= 108{,}4 \text{ kW}$

3. $\eta_{bt} = \frac{\text{brake power output}}{\text{fuel power input}}$

$= \frac{P_b \times 100}{CV \times m}$

$= \frac{90{,}32 \times 100}{40 \times 10^3 \times \frac{5{,}4}{12 \times 60}}$

$= \frac{90{,}32 \times 100 \times 12 \times 60}{40 \times 10^3 \times 5{,}4}$

$= 30{,}1\%$

4. To find the air standard efficiency (η_a), first find compression ratio (r).
Swept volume = V_s and clearance volume = 13% of V_s = 0,13 V_s.

$r = \frac{\text{swept volume + clearance volume}}{\text{clearance volume}}$

$= \frac{V_s + 0{,}13\ V_s}{0{,}13\ V_s}$

$= \frac{1{,}13}{0{,}13}$

$\therefore r = 8{,}7:1$

$\eta_a = 1 - \left(\frac{1}{r}\right)^{\gamma - 1}$

$= 1 - \frac{1}{8{,}7^{0{,}4}}$

$= 0{,}579 \text{ or } 57{,}9\%$

5.

$$\text{Relative efficiency} = \frac{\text{brake thermal efficiency}}{\text{air standard efficiency}}$$

$$= \frac{30{,}1}{57{,}9} \times 100$$

$$= 52\%$$

Example 1.6

A three-cylinder, two-stroke petrol engine, working on the constant volume (Otto) cycle, has a swept volume of 333 cm^3 per cylinder and a compression ratio of 7,5 : 1. These results were obtained from a 30-minute test on the engine, at a speed of 4 500 r/min.: indicated mean effective pressure = 388 kPa; total fuel used during test = 3,75 kg; brake torque at 4 500 r/min. = 54 Nm; heat value of fuel = 42 MJ/kg.

1. Calculate the air standard efficiency (ASE) when gamma = 1,4.

2. Calculate the indicated power (IP) in kilowatts.
3. Calculate the brake power (BP) in kilowatts.
4. Calculate the mechanical efficiency (Mη).
5. Calculate the indicated thermal efficiency (ITη).
6. Calculate the heat energy available (Q) in kJ/min.
7. Calculate the heat lost to friction (Q) in kJ/min.

Solutions

1. $\eta_a = 1 - \left(\frac{1}{r}\right)^{\gamma-1}$
$= 1 - \frac{1}{7{,}5^{0{,}4}}$
$= 0{,}554$ or $55{,}4\%$

2. $P_i = \frac{pLANEx}{60}$
$= 388 \times \frac{333}{10^6} \times \frac{4\,500}{60} \times 3$
$= 29{,}1$ kW

$p = 388$ kPa
$L \times A = \frac{333}{10^6}$
$NE = 4{,}500$ firing strokes
$x = 3$ cylinders

3. $P_b = \frac{2\pi NT}{60}$
$= 2\pi \times \frac{4\,500}{60} \times 54$
$= 25\,400$ watts
$= 25{,}4$ kW

$N = 4\,500$ revs/ min.; $T = 54$ Nm

4. $\eta_m = \frac{P_B}{P_I} \times 100$
$= \frac{25{,}4}{29{,}1} \times 100$
$= 87{,}2\%$

5. $\eta_{it} = \frac{\text{indicated power output}}{\text{fuel power input}}$
$= \frac{P_i \times 100}{H_V \times m}$

$$= \frac{29,1 \times 100 \times 30 \times 60}{42 \times 10^3 \times 3,75}$$
$$= 33,2\%$$

$H_V = 42 \times 10^3$; $m = \frac{3,75}{30 \times 60}$ kg/s

6. $Q = m \times Hv$
$= \frac{3,75}{30} \times 42 \times 10^3$
$= 5\ 250$ kJ/min

7. $Q = P_i - P_b$
$= 29,1 - 25,4$
$= 3,7$ kW
$= 3,7$ kJ/s
$= 3,7 \times 60$
$= 222$ kJ/min

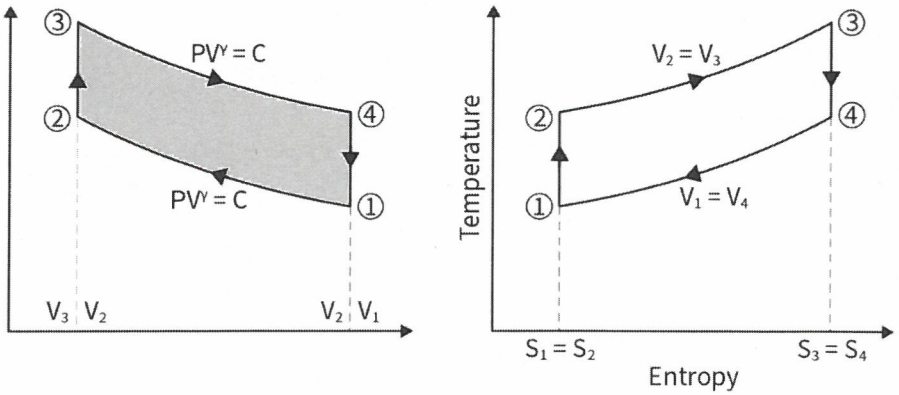


Figure 1.3

In ideal conditions, compression and expansion are adiabatic ($PV^\gamma = C$).
W per cycle = shaded area
= (area V_3; 3; 4; V_4) − (area V_2; 2; 1; V_1)
$= \left(\frac{P_3V_3 - P_4V_4}{\gamma - 1}\right) - \left(\frac{P_2V_2 - P_1V_1}{\gamma - 1}\right)$
W under curve V_2, 2, 1, V_3 would be a minus (−) quantity, and a minus × a minus would give a positive value.

So, the shaded area is under the top curve, minus the area under the bottom curve.

To do this mathematically, change $\frac{P_1V_1 - P_2V_2}{\gamma - 1}$ to $\frac{P_2V_2 - P_1V_1}{\gamma - 1}$ to get a positive value.

$$W = \left(\frac{P_3V_3 - P_4V_4}{\gamma - 1}\right) - \left(\frac{P_2V_2 - P_1V_1}{\gamma - 1}\right)$$

but $P_1V_1 = mRT_1, P_2V_2 = mRT_2, P_3V_3 = mRT_3$ and $P_4V_4 = mRT_4$

$$W = \left(\frac{mRT_3 - mRT_4}{\gamma - 1}\right) - \left(\frac{mRT_2 - mRT_1}{\gamma - 1}\right)$$

$$W/\text{cycle} = mR\left(\frac{T_3 - T_4 - T_2 + T_1}{\gamma - 1}\right) \quad \ldots\ldots\ldots\

$(1{,}785)^{\frac{1}{n-1}} = 6{,}2$

$\therefore \frac{1}{n-1} = \frac{\ln 6{,}2}{\ln 1{,}785}$

$= 3{,}15$

$1 = 3{,}15n - 3{,}15$

$3{,}15n = 4{,}15$

$n = \frac{4{,}15}{3{,}15}$

$\approx 1{,}32$

$PV^n = C$

2,58 cm^2

$PV^n = C$

≈ is the symbol for approximately.

2. 1 cm on the PV diagram = (length of 1 cm) × (height of 1 cm)

$1\ cm^2 = 680\ kPa \times 0{,}01114\ m^3$

$\therefore\ 2{,}58\ cm^2 = (680 \times 0{,}01114) \times 2{,}58$

$\therefore$ W/cycle = 19,55 kJ

Example 1.8

The volume–compression ratio for an Otto cycle engine is 9 : 1. The compression index is 1,35 and the expansion index is 1,32. At the start of compression, the gas is at 69 kPa and 100 °C. The temperature at the beginning of expansion is 1 450 K; R = 288 J/kg.K.

1. Calculate the absolute pressure (in kPa) and the temperature at all the change points.
2. Calculate the work done per kilogram of gas.
3. Calculate the mean effective pressure in kPa if the stroke = piston diameter = 153 mm.

Solutions

$R = 0{,}288\ kJ/kg.K$

$n_c = 1{,}35$

$n_e = 1{,}32$

$m = 1\ kg$

1. Find all pressures and temperatures.

$P_1 V_1^{n_c} = P_2 V_2^{n_c}$

$\therefore\ P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{n_c}$

$= 69 \times (9)^{1{,}35}$

$= 1\ 340\ kPa$

$$T_2 = \frac{P_2V_2T_2}{P_1V_1}$$
$$= \frac{1\,340}{69} \times \frac{1}{9} \times 373$$
$$= 805\ \text{K}$$

$$P_3 = \frac{V_2P_2T_3}{V_3T_2}$$
$$= \frac{1\,340 \times 1\,450}{805}$$
$$= 2\,414\ \text{kPa}$$

$$P_3V_3^{n_e} = P_3V_3^{n_e}$$
$$\therefore P_4 = P_3\left(\frac{V_3}{V_4}\right)^{n_e}$$
$$= 2\,414 \times \left(\frac{1}{9}\right)^{1,32}$$
$$= 132{,}6\ \text{kPa}$$

$$T_4 = \frac{P_4V_4T_3}{P_3V_3}$$
$$= \frac{132{,}6}{2\,414} \times \frac{9}{1} \times 1\,450$$
$$= 717\ \text{K}$$

2. Find the work done (per kg).

$$W = \left(\frac{P_3V_3 - P_4V_4}{n_e - 1}\right) - \left(\frac{P_2V_2 - P_1V_1}{n_c - 1}\right)$$
$$= \left(\frac{mR(T_3 - T_4)}{1{,}32 - 1}\right) - \left(\frac{mR(T_2 - T_1)}{1{,}35 - 1}\right)$$
$$= \left(\frac{1 \times 0{,}288(1\,450 - 717)}{0{,}32}\right) - \left(\frac{1 \times 0{,}288(805 - 373)}{0{,}35}\right)$$
$$= \frac{0{,}288 \times 733}{0{,}32} - \frac{0{,}288 \times 432}{0{,}35}$$
$$= 660 - 355$$
$$= 305\ \text{kJ/kg}$$

3. To find the mean effective pressure (MEP), first find the mass of gas per cycle (m)

$$P_1V_1 = mRT_1$$
$$\therefore m = \frac{P_1V_1}{RT_1}$$

But V_1 is unknown = SV + CV

Swept volume (SV) $= \frac{\pi}{4} \times \frac{153}{10^3} \times \frac{153}{10^3} \times \frac{153}{10^3}$

$= 0{,}00282\ \text{m}^3$

Clearance volume (CV) $= \frac{0{,}00282}{8}$ $0{,}00035\ m^3$

$V_1 = 0{,}00282 + 0{,}00035$

$= 0{,}00317\ m^3$

$\therefore m = \frac{69 \times 0{,}00317}{0{,}288 \times 373}$

$= 0{,}00204\ kg$

$p_m = \frac{W \times \text{mass per cycle}}{\text{swept volume}}$

$= \frac{305 \times 0{,}00204}{0{,}00282}$

$= 221\ kPa$

Example 1.9

A single-cylinder, four-stroke petrol engine working on the Otto cycle has a volume–compression ratio of 8,5 : 1. Polytropic index for compression and expansion = 1,4: brake power of the engine at 2 000 r/min. = 11,2 kW, with a mechanical efficiency of 83%. The cylinder diameter and stroke are equal and
the indicated mean effective pressure is 620 kPa. The pressure at the start of the compression stroke is 90 kPa.

1. Calculate the indicated power in kilowatts.
2. Calculate the work done in joules per cycle.
3. Calculate the piston diameter in millimetres.
4. Calculate the volumes in cubic metres and the pressures in kilopascal at all the main points in the cycle.

Solutions

1. Find the indicated power (P_i).

$P_i = \frac{P_b}{\eta_m}$

$= \frac{11{,}2}{83} \times 100$

$= 13{,}5\ kW$

2. Find the work done per cycle.

$W/cycle = \frac{\text{indicated power (joules/ min.)}}{\text{number of cycles/min.}}$

$= \frac{13{,}5 \times 10^3 \times 60}{\frac{2\,000}{2}}$

$= 810$ joules per cycle

3. Find the piston diameter (D).

$$P_i = \frac{pLANEx}{60}$$

$$13{,}5 = 620 \times D \times \frac{\pi}{4} D^2 \times \frac{2\,000}{60 \times 2} \times 1$$

$$D = \sqrt[3]{\frac{13{,}5 \times 4 \times 2 \times 60}{620 \times \pi \times 2\,000}}$$

$$= \sqrt[3]{0{,}001663}$$

$$= 0{,}1184 \text{ m}$$

$$= 118{,}4 \text{ mm}$$

$P_i = 13{,}5$ kW

$p = 620$kPa

$L = D$

$A = \frac{\pi}{4} D^2$

$NE = \frac{2\,000}{2}$

$x = 1$ cylinder

4. Find the volume and pressure at all the points.

$$SV = \frac{\pi}{4} \times D^2 \times D = \frac{\pi}{4} \times D^3$$

$$= \frac{\pi}{4} \times 0{,}001663$$

$$= 0{,}00131 \text{ m}^3$$

$$D^3 = 0{,}001663$$

$$CV = V_2$$

$$= \frac{0{,}00131}{7{,}5}$$

$$= 0{,}000175 \text{ m}^3$$

$$V_1 = V_4$$

$$= SV + CV$$

$$= 0{,}0001754 + 0{,}00131$$

$$= 0{,}0014845 \text{ m}^3$$

$$P_2 = P_1 \left(\frac{V_1}{V_2}\right)^n$$

$$= 90 \times (8{,}5)^{1{,}4}$$

$$= 90 \times 20$$

$$= 1\,800 \text{ kPa}$$

$$P_4 = P_3 \left(\frac{V_3}{V_4}\right)^n$$

$$= P_3 \left(\frac{1}{8{,}5}\right)^{1{,}4}$$

$$= P_3 \times \frac{1}{20}$$
$$= \frac{P_3}{20} \text{ or } P_3 = 20\ P_4$$

$$W = \frac{(P_3V_3 - P_4V_4)}{n-1} - \frac{(P_2V_2 - P_1V_1)}{n-1}$$
$$0{,}81 = \left(\frac{(20\ P_4 \times 0{,}000175) - (P_4 \times 0{,}00148)}{1{,}4-1}\right) - \left(\frac{(1\,800 \times 0{,}000175) - (90 \times 0{,}00148)}{1{,}4-1}\right)$$
$$= \left(\frac{(0{,}0035\ P_4 - 0{,}00148\ P_4)}{0{,}4}\right) - \left(\frac{(0{,}315 - 0{,}133)}{0{,}4}\right)$$
$$= \frac{0{,}00202\ P_4}{0{,}4} - \frac{0{,}182}{0{,}4}$$
$$= 0{,}00505\ P_4 - 0{,}455$$
$$= 5\ 020 \text{ kPa}$$

But $P_3 = 20\ P_4$

$$P_4 = \frac{0{,}81 + 0{,}455}{0{,}00505}$$
$$= \frac{1{,}265}{0{,}00505}$$
$$P_3 = 251 \times 20$$
$$= 5\ 020 \text{ kPa}$$

Example 1.10

The information below refers to a single-cylinder, four-stroke petrol engine working on the Otto cycle.

T_2 = 545 K; volume–compression ratio = 6,2 : 1; T_1 = 305 K. Compression and expansion follows the law $PV^n = C$ (polytropic).

The enclosed area on the P–V diagram is 2,58 cm^2. 10 mm = 680 kPa and 10 mm = 0,01114 m^3 on the P–V diagram.

The mass of air used is 0,0355 kg per cycle. R = 0,288 kJ/kg.K.

1. Calculate the compression and expansion index *n*.
2. Calculate the work done in kJ per cycle.
3. Calculate the temperature T_4 in Kelvin.
4. Calculate the temperature T_3 in Kelvin.

Solutions

1. Find the index n.

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1}$$
$$\therefore \frac{545}{305} = \left(\frac{6{,}2}{1}\right)^{n-1}$$
$$\therefore\ 1{,}785 = 6{,}2^{n-1}$$

$\therefore (1{,}785)^{\frac{1}{n-1}} = 6,2^{n-1}$

$\therefore \frac{1}{n-1} = \frac{\ln 6{,}2}{\ln 1{,}785}$

$= 3{,}15$

$n = \frac{4{,}15}{3{,}15}$

$\approx 1{,}32$

2. Find the work done per cycle.

1 cm^2 on P − V diagram = Length of 1 cm × height of 1 cm

1 cm^2 = 680 kPa × 0,01114 m^3

2,58 cm^2 = 680 × 0,01114 × 2,58

Work done/cycle = 19,55 kJ

3. Find T_4

$W/cycle = \left(\frac{P_3V_3 - P_4V_4}{n-1}\right) - \left(\frac{P_2V_2 - P_1V_1}{n-1}\right)$

$= \frac{P_3V_3 - P_4V_4 - P_2V_2 + P_1V_1}{n-1}$

$= \frac{mR}{n-1}(T_3 - T_4 - T_2 + T_1)$

$19{,}55 = \frac{0{,}0355 \times 0{,}288}{(1{,}32-1)}(T_3 - T_4 - 545 + 305)$

But $\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{n-1}$

$T_3 = T_4 \times (6{,}2)^{1{,}32-1} = T_4 \times (6{,}2)^{0{,}32}$

$= 1{,}793\ T_4$

$\frac{19{,}55 \times 0{,}32}{0{,}0355 \times 0{,}288} + 240 = T_3 - T_4$

$613 + 240 = 1{,}792\ T_4 - T_4$

$853 = 0{,}792\ T_4$

$T4 = \frac{853}{0{,}793}$

$= 1\,075$ K

4. Find T_3.

$T_3 = 1{,}792\ T_4$
$= 1{,}793 \times 1\ 075$
$= 1\ 927\ K$

Exercise 1.1

1. The relative efficiency of a two-stroke petrol engine working on the constant volume cycle = 50,6%. The volume–compression ratio = 7,5 : 1 and the mechanical efficiency = 78%. The engine uses 1,69 kg petrol per hour with a heat value of 44 MJ/kg. Assume gamma as 1,4.

Calculate the following:
1.1 air standard efficiency
1.2 brake thermal efficiency
1.3 indicated thermal efficiency
1.4 indicated power
1.5 brake power

2. A four-cylinder four-stroke (Otto cycle petrol engine with a bore of 75 mm and a stroke of 80 mm has an indicated mean effective pressure of 800 kPa at 3 000 r/min. in each cylinder. The compression ratio is 8,2 : 1. The brake thermal efficiency is 25% and the calorific value of the petrol is 41 MJ/kg. Assume a mechanical efficiency of 74%. Assume gamma as 1,4.

Calculate the following:
2.1 indicated power
2.2 brake power
2.3 fuel consumption in kg per hour
2.4 air standard efficiency
2.5 brake relative efficiency

3. An engine working on the Otto cycle has a cylinder diameter of 101,6 mm and a stroke of 152,4 mm. At the start of compression, the air temperature is 373 K and pressure is 68,95 kPa. The volume–compression ratio of the engine is 8,5 : 1. The highest pressure in the cycle = 3 275 kPa. Index of compression and expansion = 1,25 and the gas constant = 0,288 kJ/kg.K. Assume C_v as 0,718 kJ/kgK.

Calculate the following:
3.1 pressures after compression and expansion
3.2 cylinder and clearance volumes in liters
3.3 temperatures at all the main points
3.4 work done
3.5 heat supplied at constant volume

4. An engine working on the constant volume cycle has 4 cylinders, and operates on the four stroke (Otto principle at 3 000 revs/min. The engine has a compression ratio of 8,6 : 1 and develops an indicated power of 30 kW with an indicated mean effective pressure = 700 kPa. Compression and expansion index n = 1,3. The piston diameter and stroke are equal. The pressure at the beginning of the compression stroke = 90 kPa and the highest pressure in the cycle = 4 400 kPa. The temperature at start of compression is 350 K.

Calculate the following:
4.1 piston diameter
4.2 volumes at all the main points in cm^3
4.3 pressures after compression and after expansion
4.4 work done
4.5 temperatures at all the main points in the cycle

3 to 4: Adiabatic expansion according to the law $PV^{\gamma} = C$; pressure decreases from P_3 to P_4; temperature decreases from T_3 to T_4; volume increases from V_3 to V_4; entropy remains constant at $S_3 - S_4$.

4 to 1: Heat rejected at constant volume; volume remains constant at $V_4 = V_1$; temperature decreases from T_4 to T_1; pressure decreases from P_4 to P_1; entropy decreases from S_4 to S_1.

This process completes the cycle and returns the gas to its original state.

How to find the actual work done per cycle

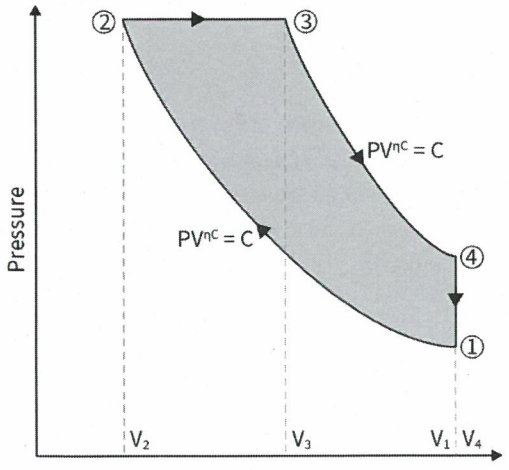


Figure 1.4

Work done per cycle = shaded area

$$= (\text{area } V_2, ②, ③, V_3) + (\text{area } V_3, ③, ④, V_4) - (\text{area } V_2, ②, ①,$$
$$= P_2 \text{ or } P_3(V_3 - V_2) + \left(\frac{P_3V_3 - P_4V_4}{ne-1}\right) - \left(\frac{P_2V_2 - P_1V_1}{nc-1}\right)$$
$$= P_3V_3 - P_2V_2 + \left(\frac{P_3V_3 - P_4V_4}{ne-1}\right) - \left(\frac{P_2V_2 - P_1V_1}{nc-1}\right)$$
$$= mR(T_3 - T_2) + \left(\frac{mR(T_3 - T_4)}{ne-1}\right) - \left(\frac{mR(T_2 - T_1)}{nc-1}\right)$$
$$= mR\left[(T_3 - T_2 + \left(\frac{T_3 - T_4}{ne-1}\right) - \left(\frac{T_2 - T_1}{nc-1}\right)\right] \text{ joules per cycle}$$

If the index for compression and expansion are the same values:

$$W/\text{cycle} = mR\left[(T_3 - T_2) + \left(\frac{T_3 - T_4 - T_2 + T_1}{n-1}\right)\right] \text{joules/cycle,}$$

where T_4 and T_2 are adiabatic temperatures

If the cycle is adiabatic:

$$W/\text{cycle} = mR\left[(T_3 - T_2) + \left(\frac{T_3 - T_4' - T_2' + T_1}{\gamma - 1}\right)\right] \text{joules/cycle}$$

Heat added at constant pressure $= m \times C_P(T_3 - T_2)$ joules

Heat added at constant pressure $= m \times C_V(T_4 - T_1)$ joules

How to find the air standard efficiency (ASE) of a diesel cycle

$$\eta_a = \frac{\text{heat received} - \text{heat rejected}}{\text{heat received}}$$
$$= 1 - \frac{\text{heat rejected}}{\text{heat received}}$$
$$= 1 - \frac{m \times C_V(T_4 - T_1)}{m \times C_P(T_3 - T_2)}$$

But $\frac{C_P}{C_V} = \gamma$

$$\frac{C_V}{C_P} = \frac{1}{\gamma}$$
$$\eta_a = 1 - \frac{1}{\gamma}\left(\frac{T_4 - T_1}{T_3 - T_2}\right) \quad \ldots\ldots\ldots\

Solutions

1. Swept volume $= \frac{\pi D^2}{4} \times$ stroke

$$= \frac{\pi}{4} \times \frac{152,4}{10^3} \times \frac{152,4}{10^3} \times \frac{228,6}{10^3}$$

$$= 0,00418 \text{ m}^3$$

Compression–expansion ratio $= \frac{SV+CV}{CV}$

CV = clearance volume;
SV = swept volume

$15 \times CV = 0,00418 + CV$

$CV = \frac{0,00418}{14}$

$= 0,000298 \text{ m}^3$

$V_2 = 0,000298 \text{ m}^3$

$V_1 = 0,00418 + 0,000298 = 0,004478 \text{ m}^3$

V_3 = (10% of swept volume) + (clearance volume)

$= \left(\frac{10}{100} \times 0,0418\right) + 0,000298$

$= 0,000418 + 0,000298$

$= 0,000716 \text{ m}^3$

Find P_2

$(P_1V_1)^{1,32} = (P_2V_2)^{1,32}$

$\therefore P_2 = P_1\left(\frac{V_1}{V_2}\right)^{1,32}$

$= 103,4 \times (15)^{1,32}$

$= 3\,690$ kPa

Find P_4

$(P_3V_3)^{1,32} = (P_4V_4)^{1,32}$

$\therefore P_4 = P_3\left(\frac{V_3}{V_4}\right)^{1,32}$

$= 3\,690\left(\frac{0,000716}{0,004478}\right)^{1,32}$

$= 328$ kPa

Find T_2

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$\therefore T_2 = \frac{P_2V_2T_1}{P_1V_1}$

$$= \frac{3\,690 \times 10^3 \times 0{,}000298 \times 373}{103{,}4 \times 10^3 \times 0{,}004478}$$

$$= 890\ \text{K}$$

$$\text{or } \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{1}}$$

$$T_2 = 890\ \text{K}$$

2. Find the work done (in kilojoules per cycle).

$$W = P_2(V_3 - V_2) + \left(\frac{P_3V_3 - P_4V_4}{(n-1)}\right) - \left(\frac{P_2V_2 - P_1V_1}{(n-1)}\right)$$

$$= 3\,690(0{,}000716 - 0{,}000298) + \left(\frac{(3\,690 \times 0{,}000716) - (331 \times 0{,}004478)}{(1{,}32 - 1)}\right) - \left(\frac{(3\,690 \times 0{,}000298) - (103{,}4 \times 0{,}004478)}{(1{,}32 - 1)}\right)$$

$$= [3\,690 \times 0{,}000418] + \left(\frac{2{,}64 - 1{,}48}{0{,}32}\right) - \left(\frac{1{,}1 - 0{,}463}{0{,}32}\right)$$

$$= 1{,}54 + 3{,}63 - 1{,}99 = 3{,}18\ \text{kJ}$$

$$\text{or } W = (P_3V_3 - P_4V_4) + \frac{(P_3V_3 - P_4V_4)}{n-1} - \frac{(P_2V_2 - P_1V_1)}{n-1}$$

$$= mR(T_3 - T_2) + \frac{mR(T_3 - T_4)}{n-1} - \frac{mR(T_2 - T_1)}{n-1}$$

Find the mass of the gas.

$$P_1V_1 = mRT_1$$

$$\therefore m = \frac{P_1V_1}{RT_1}$$

$$= \frac{103{,}4 \times 10^3 \times 0{,}004478}{0{,}288 \times 10^3}$$

$$= 0{,}00431\ \text{kg}$$

$$N = 0{,}00431 \times 288(2\,140 - 890) + \left(\frac{0{,}00431 \times 288(2\,140 - 1\,190)}{(1{,}32-1)}\right) - \left(\frac{0{,}00431 \times 288(890 - 373)}{(1{,}32-1)}\right)$$

$$= 0{,}00431 \times 288 \times 1\,250 + \left(\frac{0{,}00431 \times 288 \times 950}{0{,}32}\right) - \left(\frac{0{,}00431 \times 288 \times 517}{0{,}32}\right)$$

$$= 1\,552 + 3\,531 + 2\,000\ \text{J}$$

$$= 3\,180\ \text{J}$$

$$= 3{,}18\ \text{kJ}$$

3.

$$Q = m \times C_P \times (T_3 - T_2)\text{J}$$

$= 0{,}00431 \times 0{,}997 \times 10^3\ (2\,140 - 890)$
$= 0{,}00431 \times 0{,}997 \times 10^3 \times 1\,250$
$= 5\,370$ J
$= 537$ kJ

$m = 0{,}00431$ kg

Example 1.12

An engine working on the diesel cycle has a single cylinder with a diameter of 222 mm and a stroke of 317,5 mm. The volume–compression ratio is 15 : 1 and the volume–expansion ratio is 6,25 : 1. $P_4 = 276{,}8$ kPa, $T_4 = 1\,125$ K.

The index for compression and expansion curves is 1,38.

1. Calculate the volume in m3, the pressure in kPa and the absolute temperature in Kelvin at each main point in the cycle.

2. Calculate the work done in kilojoules per 0,0113 kg of air per cycle. Assume R for air = 0,288 kJ/kg.K.

Solutions

1.

Swept volume (SV) $= \frac{\pi D^2}{4} \times$ stroke
$= \frac{\pi}{4} \times \left(\frac{222}{10^3}\right)^2 \times \frac{317{,}5}{10^3}$
$= 0{,}0123\ m^3$

$V_2 = \frac{SV}{14}$
$= \frac{0{,}0123}{14}$
$= 0{,}000877\ m^3$

$V_1 = V_4$
$= SV + CV$
$= 0{,}0123 + 0{,}000877$
$= 0{,}01317$
$\approx 0{,}0132\ m^3$

Clearance volume $= CV$

Find V_3

$V_3 \times \text{exp a nsion ratio} = V_2 \times \text{compression ratio}$

$V_3 = V_2 \times \left(\frac{\text{compression ratio}}{\text{exp ansion ratio}}\right)$

$= 0{,}000877 \times \left(\frac{15}{6{,}25}\right)$

$= 0{,}00211\ m^3$

Find the pressures P_1, P_2 and P_3

$P_2 = P_3$

$P_3 V_3^{\ n} = P_4 V_4^{\ n}$

$\therefore\ P_3 = P_4 \left(\frac{V_4}{V_3}\right)^n$

$= 276{,}8 \left(\frac{0{,}0132}{0{,}00211}\right)^{1{,}38}$

$= 276{,}8 \times 6{,}25^{1{,}38}$

$= 3\ 470\ kPa$

$P_1 V_1^{\ n} = P_2 V_2^{\ n}$

$\therefore P_1 = P_2 \left(\frac{V_2}{V_1}\right)^n$

$= 3\ 470 \left(\frac{0{,}000877}{0{,}013177}\right)^{1{,}38}$

$= 82{,}6\ kPa$

Find the temperatures T_1, T_2 and T_3

$\frac{P_3 V_3}{T_3} = \frac{P_4 V_4}{T_4}$

$\therefore\ T_3 = \frac{P_3 V_3 T_4}{P_4 V_4}$

$= \frac{3\ 470 \times 0{,}00211 \times 1\ 125}{176{,}8 \times 0{,}0132}$

$= 3\ 530\ K$

$\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3}$

$\therefore\ T_2 = \frac{P_2 V_2 T_2}{P_3 V_3}$

$= \frac{3\ 470 \times 0{,}000877 \times 3\ 530}{3\ 470 \times 0{,}00211}$

$= 1\ 457\ K$

$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

$$\therefore T_1 = \frac{P_1V_1T_1}{P_2V_2}$$
$$= \frac{82{,}6 \times 0{,}0132 \times 1\,467}{3\,470 \times 0{,}000877}$$
$$= 526\ \text{K}$$

2.

$$W = P_2(V_3 - V_2) + \left(\frac{P_3V_3 - P_4V_4}{(n-1)}\right) - \left(\frac{P_2V_2 - P_1V_1}{(n-1)}\right)$$
$$= (P_3V_3 - P_2V_2) + \left(\frac{P_3V_3 - P_4V_4}{(n-1)}\right) - \left(\frac{P_2V_2 - P_1V_1}{(n-1)}\right)$$
$$= mR(T_3 - T_2) + \left(\frac{mR(T_3 - T_4)}{(n-1)}\right) - \left(\frac{mR(T_2 - T_1)}{(n-1)}\right)$$
$$= mR\left[(T_3 - T_2) + \left(\frac{T_3 - T_4 - T_2 - T_1}{(n-1)}\right)\right]$$
$$= 0{,}0113 \times 0{,}288 \times 10^3\left[(3\,530 - 1\,467) + \left(\frac{3\,530 - 1\,125 - 1\,467 + 526}{1{,}38 - 1}\right)\right]$$
$$= 3{,}25\left[2\,066 + \left(\frac{1\,464}{0{,}38}\right)\right]$$
$$= 3{,}25(2\,066 + 2\,863)\ \text{J}$$
$$= 19{,}2\ \text{kJ}$$

Example 1.13

Information on an engine working on the diesel cycle of operations: stroke volume = 0,0043 m^3 and volume-compression ratio = 16 : 1 R = 0,288 kJ/kg.K

- T_1 = 373 K; P_1 = 103,4 kPa
- P_4 = 310,2 kPa
- C_v = 0,712 kJ/kg.K
- Compression and expansion index = 1,32

1. Calculate the highest pressure in kPa.
2. Calculate the volumes and temperatures at all the main points in the cycle.
3. Calculate the heat flow through the cylinder wall during one expansion stroke (3 to 4).

Assume the mass of air per cycle = 0,008 kg.

Solutions

1. Find P_3 and P_2

$$P_1V_1^{\,n} = P_2V_2^{\,n}$$
$$\therefore P_2 = P_1\left(\frac{V_1}{V_2}\right)^n$$
$$= 103{,}4 \times (16)^{1{,}32}$$
$$= 4\,015\ \text{kPa}$$

2. Find the volumes V_1, V_2, V_3 and V_4

Compression ratio $= \frac{SV+CV}{CV}$

$16 \times CV = 0{,}0043 + CV$

$V_2 = 0{,}000287\ m^3$

$V_1 = 0{,}0043 + 0{,}000287$

$= 0{,}004587\ m^3$

$SV = 0{,}0043\ m^3;\ CV = \frac{0{,}0043}{15} = 0{,}000287\ m^3$

Find V_3

$P_3 V_3^{\ n} = P_4 V_4^{\ n}$

$\therefore\ P_3 = \left(\frac{P_4}{P_3}\right)^{\frac{1}{n}} \times V_4$

$= \left(\frac{310{,}2}{4\,015}\right)^{\frac{1}{1{,}32}} \times 0{,}004587$

$= 0{,}00066\ m^3$

Find the temperatures T_2, T_3 and T_4

$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

$\therefore\ T_2 = \frac{P_2 V_2 T_1}{P_1 V_1}$

$= \frac{4\,015 \times 0{,}000287 \times 373}{103{,}4 \times 0{,}004587}$

$= 905\ K$

or

$\frac{T_2}{T_1} = \left(\frac{P_2}{P_2}\right)^{\frac{n-1}{n}}$

$= 373\left(\frac{4\,015}{103{,}4}\right)^{\frac{0{,}32}{1{,}32}}$

$= 905\ K$

$\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3}$

$\therefore\ T_3 = \frac{P_2 V_3 T_2}{P_2 V_2}$

$= \frac{0{,}00066 \times 905}{0{,}000287}$

$= 2\,081\ K$

$\frac{P_3 V_3}{T_3} = \frac{P_4 V_4}{T_4}$

$$\therefore T_4 = \frac{P_4V_4T_3}{P_3V_3}$$
$$= \frac{310{,}2 \times 0{,}004587 \times 2\,081}{4\,015 \times 0{,}00066}$$
$$= 1\,118\ \text{K}$$

3. Find the heat flow (Q).

$$Q = (\text{work done}) + (\text{change in internal energy})$$
$$= W + \Delta U$$
$$= \left(\frac{P_3V_3 - P_4V_4}{(n-1)}\right) + [m \times C_V(T_4 - T_3)]\ \text{J}$$
$$= \left(\frac{mR(T_3 - T_4)}{n-1}\right) + [0{,}008 \times 0{,}712 \times 10^3(1\,118 - 2\,081)]$$
$$= \frac{0{,}008 \times 0{,}288 \times 10^3(2\,081 - 1\,118)}{1{,}32 - 1} + (0{,}008 \times 712 \times -963)$$
$$= \left(\frac{0{,}008 \times 288 \times 963}{0{,}32}\right) - 5\,480$$
$$= 6\,934 - 5\,485$$
$$= 1\,449\ \text{J}$$
$$= 1{,}449\ \text{kJ}$$

Example 1.14

Information on an engine working on the diesel cycle of operations: stroke volume = 0,0043 m^3 and volume-compression ratio = 16 : 1 R = 0,288 kJ/kg.K

- T_1 = 373 K; P_1 = 103,4 kPa
- P_4 = 310,2 kPa
- C_v = 0,712 kJ/kg.K
- Compression and expansion index = 1,32

1. Calculate the highest pressure in kPa.
2. Calculate the volumes and temperatures at all the main points in the cycle.
3. Calculate the heat flow through the cylinder wall during one expansion stroke (3 to 4).

Assume the mass of air per cycle = 0,008 kg.

Solutions

1. Find the volumes V_1, V_3 and V_4 in cm^3.

$$\text{Swept volume} = \frac{\pi D^2}{4} \times \text{stroke}$$
$$= \frac{\pi}{4} \times 10 \times 10 \times 10{,}8$$
$$= 850\ \text{cm}^3$$

$V_1 = SV + CV$
$= 850 + 50$
$= 900\ cm^3$

$CV = 50\ cm^3$

Now $V_3 = (10\%\ of\ SV) + CV$
$= \left(\frac{10}{100} \times 850\right) + 50 = 85 + 50 = 135\ cm^3$

Find the pressures P_2, P_3 and P_4 and the temperatures T_2, T_3 and T_4.

$P_1 {V_1}^n = P_2 {V_2}^n$
$\therefore\ P_2 = P_1 \left(\frac{V_1}{V_2}\right)^n$
$= 103 \left(\frac{900}{50}\right)^{1,32}$
$= 4\ 670\ kPa$

$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$
$\therefore\ T_2 = \frac{P_2 V_2 T_1}{P_1 V_1}$
$= \frac{46,7 \times 50 \times 350}{1,03 \times 900}$
$= 882\ K$

or

$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$
$= 350 \left(\frac{46,7}{1,03}\right)^{\frac{0,32}{1,32}}$
$= 882\ K$

$\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3}$
$\therefore\ T_3 = \frac{V_3 T_2}{V_2}$
$= \frac{135 \times 882}{50}$
$= 2\ 380\ K$

$P_3V_3{}^n = P_4V_4{}^n$

$\therefore\ P_4 = P_3\left(\frac{V_3}{V_4}\right)^n$

$= 4\ 670\left(\frac{135}{900}\right)^{1,32}$

$= 383\ \text{kPa}$

$\frac{P_3V_3}{T_3} = \frac{P_4V_4}{T_4}$

$\therefore\ T_4 = \frac{P_4V_4T_3}{P_3V_3}$

$= \frac{383\times900\times2\ 380}{4\ 670\times135}$

$= 1\ 300\ \text{K}$

Or

$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{\frac{n-1}{n}}$

$= 2\ 380\left(\frac{383}{4\ 670}\right)^{\frac{0,32}{1,32}}$

$= 1\ 300\ \text{K}$

2.

$P_1V_1 = mRT_1$

$\therefore\ m = \frac{P_1V_1}{RT_1} = \frac{103\times10^3\times900}{287\times350\times10^3}$

$= 0,000922\ \text{kg}$

3. Find the work done between V_3 and V_4 in kJ.

$W = \frac{P_3V_3-P_4V_4}{n-1}$

but P_3V_3

$= mRT_3$ and $P_4V_4 = mRT_4$

$W = \frac{mR(T_3-T_4)}{n-1}$

$= \frac{0,000922\times287(2\ 380-1\ 300)}{1,32-1}$

$= \frac{0,000922\times287\times1\ 080}{0,32}$

$= 894\ \text{J}$

$= 0,894\ \text{kJ}$

or $W = \frac{P_3V_3 - P_4V_4}{n-1}$

$= \frac{(46,7\times10^5\times\frac{135}{10^6}) - (3,83\times10^5\times\frac{900}{10^6})}{(1,32-1)}$

$$= \frac{631 - 345}{0{,}32}$$
$$= \frac{286}{0{,}32}$$
$$= 894 \text{ J}$$
$$= 0{,}894 \text{ kJ}$$

Exercise 1.2

1. A gas mixture is subjected to the diesel cycle with adiabatic compression from 103 kPa. The volume changes from 2 m^3 to a volume of 0,2 m^3 Initial temperature = 10 °C. Heat is added until a volume of 0,35 m^3, followed by adiabatic expansion back to the original volume. Heat is then rejected at constant volume to its original pressure. Assume gamma as 1,414. Assume C_p = 1 kJ/kg.K and C_v = 0,707 kJ/kg.K

Calculate the following:
1.1 pressures at all the main points
1.2 temperatures at all the main points
1.3 work done
1.4 mean effective pressure

2. This information refers to an engine working on the diesel cycle:
clearance volume = 282 cm^3; highest pressure in cycle = 4 650 kPa; volume–compression ratio = 15 : 1; temperature at start of compression = 373 K; gas constant = 287 J/kg.K; index for compression and expansion curves = 1,32; C_v = 718 J/kg.K; pressure at start of heat rejection = 345 kPa.

Calculate the following:
2.1 pressure at beginning of compression
2.2 cylinder volume in liters
2.3 temperatures after polytropic compression and expansion
2.4 volume and temperature after heat addition
2.5 heat supply at constant pressure

3. Pressure and temperature at the beginning of compression in a diesel cycle are 103 kPa and 100 °C respectively. Compression and expansion follows the law $PV^{1,32}$ = constant; volume–compression ratio = 20 : 1; clearance volume = 32,8 cm^3 and fuel cut-off is at 10% of stroke volume after TDC.

Calculate the following:
3.1 volume at all the main points in cm^3
3.2 pressure at all points in cycle
3.3 temperature at all points in cycle

4. A single cylinder engine operating on the diesel cycle has a clearance volume of 410 cm^3; piston diameter = 176 mm; stroke = 254 mm; fuel cut-off at 10% of swept volume after TDC; P_4 = 337,8 kPa; T_4 = 995 K; R = 0,288 kJ/kg.K; compression and expansion follows the law $PV^{1,35} = C$.

Calculate the following:
4.1 cylinder volume and volume after heat addition in liters
4.2 pressure at all main points
4.3 temperature at all main points
4.4 work done
4.5 mean effective pressure

The dual combustion cycle

This cycle is used in some modern oil engines. It is a combination of the Otto and diesel cycles as fuel is partly burnt at constant volume and partly at constant pressure, hence the name dual-combustion cycle (or composite cycle).

In any diesel engine running at more than, for example, 350 r/min., it is necessary to start injection of the fuel oil before the air temperature is sufficient to ignite it. This is because the operation of injection is essential to deliver the required quantity, and the injected oil is at atmospheric temperature and has first to be warmed up by the air in the cylinder before it ignites.

When ignition does occur, there is a rapid rise in pressure at approximately constant volume due to the heat release from the oil, which has accumulated during the early injection period. The oil that is injected after ignition, burns at approximately constant pressure.

The P–V and T–S diagrams of the dual combustion cycle are shown below.

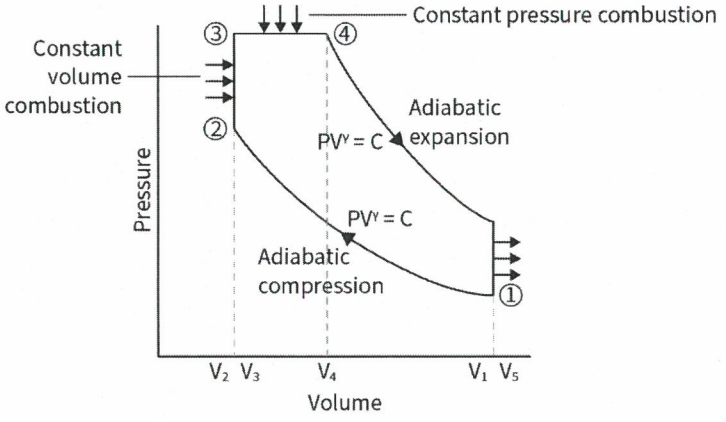


Figure 1.5

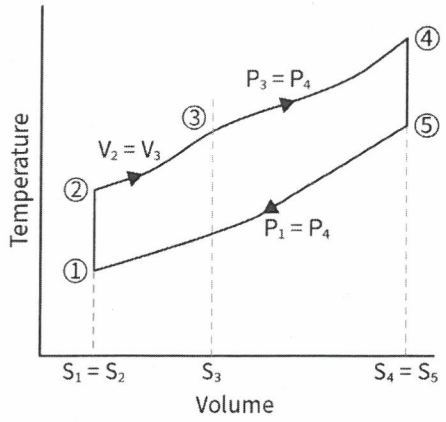


Figure 1.6

Arrangement of the cycle of the T–S diagram

1 to 2

Adiabatic compression is according to the law $PV^{\gamma} = C$. Pressure increases from P_1 to P_2 Temperature increases from T_1 to T_2 Volume decreases from V_1 to V_2 Entropy remains constant at

$S_1 = S_2$

2 to 3

Heat is added at constant volume. Volume remains constant at $V_2 = V_3$ Pressure increases from P_2 to P_3 Temperature increases from T_2 to T_3 Entropy increases from S_2 to S_3

3 to 4

Heat is added at constant pressure. Pressure remains constant at $P_3 = P_4$ Volume increases from V_3 to V_4 Temperature increases from T_3 to T_4 Entropy increases from S_3 to S_4

4 to 5

Adiabatic expansion is according to the law $PV^{\gamma} = C$. Pressure decreases from P_4 to P_5 Temperature decreases from T_4 to T_5 Volume increases from V_4 to V_5
Entropy remains constant at $S_4 = S_5$

5 to 1

Heat is rejected at constant volume. Volume remains constant at $V_5 = V_1$ Pressure decreases from P_5 to P_1 Temperature decreases from T_5 to T_1 Entropy decreases from S_5 to S_1
This process completes the cycle and returns the gas to its original state.

Air standard efficiency (ASE) of the dual cycle

Total heat supplied = heat supplied during ② to ③ + heat supplied during ③ to ④

Heat supplied = $C_V(T_3 - T_2) + C_P(T_4 - T_3)$ joules

Heat rejected = heat rejected during ⑤ to ①

$Q = C_V(T_5 - T_1)$ joules

$$\therefore \eta_a = \frac{\text{heat supplied} - \text{heat rejected}}{\text{heat supplied}}$$

$$= \frac{[C_V(T_3-T_2)+C_P(T_4-T_3)]-[C_V(T_5-T_1)]}{C_V(T_3-T_2)+C_P(T_4-T_3)}$$

$$= 1 - \frac{C_V(T_5-T_1)}{C_V(T_3-T_2)+C_P(T_4-T_3)}$$

$$\text{But } \frac{C_P}{C_V} = \gamma$$

$$= 1 - \left(\frac{(T_5-T_1)}{(T_3-T_2)+\gamma(T_4-T_3)}\right) \quad \ldots\ldots\ldots\

$$W/cycle = mR\left[(T_4 - T_3) + \left(\frac{(T_4 - T_5 - T_2 + T_1)}{n-1}\right)\right] \text{ joules/cycle}$$

If polytropic compression and expansion indices are not the same:

$$W/cycle = mR\left[(T_4 - T_3) + \left(\frac{T_4 - T_5}{n_e - 1}\right) - \left(\frac{T_2 + T_1}{n_c - 1}\right)\right] \frac{\text{joules}}{\text{cycle}}$$

n_c = compression index

n_e = expansion index

Heat added at constant volume (② to ③) = $m \times C_V(T_3 - T_2)$ J.

Heat added at constant pressure (③ to ④) = $m \times C_P(T_4 - T_3)$ J.

Heat rejected at constant volume (⑤ to ①) = $m \times C_V(T_5 - T_1)$ J.

Example 1.15

A single-cylinder engine working on the dual-cycle principle has a cylinder diameter of 78 mm and a stroke of 94 mm. The clearance volume is ^{1} of the swept volume.
Air at the start of compression is at a pressure of 69 kPa and 310 K. Compression and expansion index = 1,32. R = 287 J/kg.K; C_p = 1 005 J/kg.K. 0,185 kilojoule of heat is added during constant volume combustion per cycle. The pressure at point 5 in the cycle is 326 kPa.

1. Calculate the swept volume (SV) in cubic metre.
2. Calculate the volume in m^3 at each point in the cycle.
3. Calculate the pressures in kPa, temperatures in Kelvin, and the mass of air used per cycle in kilogram.

Solutions

1. $SV = \frac{\pi D^2}{4} \times \text{stroke}$

$= \frac{\pi}{4} \times \frac{78}{10^3} \times \frac{78}{10^3} \times \frac{94}{10^3}$

$= 0{,}00045 \text{ m}^3$

2.

$V_2 = V_3$

$= \frac{0{,}00045}{16}$

$0{,}000028 \text{ m}^3$

$V_1 = V_5$

$= CV + SV$
$= 0{,}00045 + 0{,}000028$
$= 0{,}000478\ m^3$

3. Find the pressure, temperature and mass of air.

$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{1{,}32}$
$= 69 \times 17^{1{,}32}$
$= 2\ 900\ kPa$

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$
$\therefore\ T_2 = \frac{P_2V_2T_1}{P_1V_1}$
$= \frac{2\ 900 \times 0{,}000028 \times 310}{69 \times 0{,}000478}$
$= 762\ K$

$P_1V_1 = mRT_1$
$\therefore\ m = \frac{P_1V_1}{RT_1}$
$= \frac{69 \times 10^3 \times 0{,}000478}{287 \times 310}$
$= 0{,}000371\ kg$

$Q = m \times C_V(T_3 - T_2)$

$0{,}185 = 0{,}000371 \times 718(T_3 - 762)$
$0{,}185 \times 10^3 = (0{,}266\ T_3) - 203$

$185 + 203 = 0{,}266\ T_3$

$\therefore\ T_3 = \frac{388}{0{,}266}$
$= 1\ 460\ K$

$\frac{P_2V_2}{T_2} = \frac{P_3V_3}{T_3}$
$\therefore\ P_3 = \frac{V_2P_2T_3}{V_3T_2}$
$= \frac{2\ 900 \times 1\ 460}{762}$
$= 5\ 550\ kPa$

$$P_4V_4{}^n = P_5V_5{}^n$$

$$\therefore V_4 = V_5\left(\frac{P_5}{P_4}\right)^{\frac{1}{n}}$$

$$= 0{,}000478\left(\frac{326}{5\,550}\right)^{\frac{1}{1{,}32}}$$

$$= 0{,}0000556\ m^3$$

$$\frac{P_3V_3}{T_3} = \frac{P_4V_4}{T_4}$$

$$\therefore T_4 = \frac{P_4V_4T_3}{P_3V_3}$$

$$= \frac{0{,}0000556 \times 1\,460}{0{,}000028}$$

$$= 2\,900\ K$$

Example 1.16

This information refers to an engine working on the dual-cycle principle: Volume–compression ratio = 16 : 1

$T_1 = 373$ K; $T_2 = 984$ K; $T_3 = 1\,240$ K; $T_5 = 1\,800$ K

Clearance volume = 0,0283 m^3. Fuel is cut off at 10% of the stroke volume after TDC (top-dead-centre). Compression and expansion are not adiabatic.

R = 0,288 kJ/kg.K; C_p = 1,006 kJ/kg.K; C_v = 0,718 kJ/kg.K; P_1 = 103,4 kPa.

1. Calculate V_4 in cubic metre.
2. Calculate the compression index.
3. Calculate P_2, P_3, P_4 and P_5 in (kPa) and T_4 in Kelvin.
4. Calculate the expansion index.
5. Calculate the heat received during combustion in kilojoule per cycle.

Solutions

1. Find V_4

$$V_2 = 0{,}0283\ m^3$$
$$SV = 0{,}0283 \times 15$$
$$= 0{,}4245\ m^3$$

$V_1 = V_5$
$= 4{,}4245 + 0{,}0283$
$= 0{,}4528\ m^3$

$V_4 = (10\%\ of\ SV) + CV$

$= \left(\frac{10}{100}\right) \times 0{,}4245 + 0{,}0283$
$= 0{,}07075\ m^3$

2. Find the compression index (n_c).

$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n_c - 1}$
$\frac{984}{373} = 16^{n_c - 1}$
$\therefore\ 2{,}638 = 16^{n_c - 1}$
$\therefore\ 2{,}638^{\frac{1}{n_c - 1}} = 16$

$\therefore\ \frac{1}{n_c - 1} = \frac{\ln 16}{\ln 2{,}638} = 2{,}86$
$\therefore\ n_c = 1{,}35$

3. Find P_2; P_3; P_4; P_5 and T_4

$P_1 V_1^{\,n_c} = P_2 V_2^{\,n_c}$
$\therefore\ P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{n_c}$
$= 103{,}4 \times (16)^{1{,}35}$
$= 4\ 360\ kPa$

$\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3}$
$\therefore\ P_3 = \frac{P_2 T_3}{T_2}$
$= \frac{4\ 360 \times 1\ 240}{984}$
$= 5\ 500\ kPa$

$P_3 = P_4$
$= 5\ 500\ kPa$

$\frac{P_1 V_1}{T_1} = \frac{P_5 V_5}{T_5}$
$\therefore\ P_5 = \frac{V_1 P_1 T_5}{V_5 T_1}$

$= \frac{103,4 \times 1\,800}{373}$

$500\ \text{kPa}$

$\frac{P_4V_4}{T_4} = \frac{P_5V_5}{T_5}$

$\therefore\ T_4 = \frac{P_4V_4T_5}{P_5V_5}$

$= \frac{5\,500 \times 0,07075 \times 1\,800}{500 \times 0,4528}$

$= 3\,092\ \text{K}$

4. Find the expansion index (n_e)

$P_4V_4^{n_e} = P_5V_5^{n_e}$

$\therefore\ \frac{P_4}{P_5} = \left(\frac{V_5}{V_4}\right)^{n_e}$

$\frac{5\,500}{500} = \left(\frac{0,4528}{0,07075}\right)^{n_e}$

$\therefore\ 11 = 6,4^{n_e}$

$\therefore\ n_e = \frac{\ln 11}{\ln 6,4}$

$= 1,291$

5. Find the heat received during combustion (Q)

Total heat – (heat at constant volume) + (heat at constant pressure)

$Q = [m \times C_V(T_3 - T_2)] + [m \times C_P(T_4 - T_3)]$

Find the mass of the air (m)

$P_1V_1 = mRT_1$

$\therefore\ m = \frac{P_1V_1}{RT_1}$

$= \frac{103,4 \times 10^3 \times 0,4528}{0,288 \times 10^3 \times 373}$

$= 0,436\ \text{kg}$

If $P_2V_2 = mRT_2$ or $P_3V_3 = mRT_3$ or $P_4V_3 = mRT_4$ or $P_5V_5 = mRT_5$ is used, the mass of air in each case should be 0,436 kg.

$Q = [0,436 \times 0,718 \times 10^3(1\,240 - 984)] + [0,436 \times 1,005 \times 10^3(3\,092 + 1\,240)]$

$= 801\,400 + 811\,509$

$= 891\,649\ J$

$= 891,649\ kJ$

$$\frac{P_4V_4}{T_4} = \frac{P_5V_5}{T_5}$$

$$\therefore T_5 = \frac{P_5V_5T_4}{P_4V_4}$$

$$= \frac{3{,}26 \times 0{,}000478 \times 2\,900}{55{,}5 \times 0{,}0000556}$$

$$= 1\,464 \text{ K}$$

or

$$T_5 = \frac{V_5P_5T_1}{V_1P_1}$$

$$= \frac{326 \times 310}{69}$$

$$= 1\,464 \text{ K}$$

Example 1.17

An engine working on the ideal dual-combustion cycle, has a compression ratio of 15 : 1. Fuel cut-off takes place at 3,5% of the stroke.

The pressure at the beginning of combustion is 4 585 kPa and pressure during constant pressure

combustion = 5 516 kPa.

Temperature at beginning of compression = 100 °C; gamma = 1,4. Assume compression and expansion is adiabatic.

1. Calculate the pressure and temperature at each main point.
2. Calculate the work done per kilogram of air per cycle if R = 288 J/kg.K.
3. Calculate the heat rejected, per kilogram of air, during exhaust stroke if C_p = 1 000 J/kg.K.

Solutions

1. Find V_4 if fuel cut-off ratio is $= \frac{V_4}{V_3}$.

$$CR = \frac{SV+CV}{CV}$$

$$= \frac{SV+1}{1}$$

$$15 = SV + 1$$

$$SV = 14$$

Fuel cut-off ratio

$= (3{,}5\%\ \text{of SV}) + \text{CV}$
$= \left(\frac{3{,}5}{100} + 14\right) + 1$
$= 0{,}49 + 1$
$= 1{,}49 : 1\ \left(\frac{V_4}{V_3}\right)$

Find P_1

$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$
$\therefore\ P_1 = P_2 \left(\frac{V_2}{V_1}\right)^{\gamma}$
$= 4\,585 \left(\frac{1}{50}\right)^{1{,}4}$
$= 103{,}5\ \text{kPa}$

Find T_2.

$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$
$\therefore\ T_2 = \frac{P_2 V_2 T_1}{P_1 V_1}$
$= \frac{4\,585}{103{,}5} \times \frac{1}{15} \times 373$
$= 1\,100\ \text{K}$

or

$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}}$
$= 373 \left(\frac{4\,585}{103{,}5}\right)^{\frac{0{,}4}{1{,}4}}$
$= 1\,100\ \text{K}$

Find T_3
$\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3}$
$\therefore\ T_3 = \frac{P_3 T_2}{P_2}$
$= \frac{5\,516 \times 1\,100}{4\,585}$
$= 1\,325\ \text{K}$

Find T_4
$\frac{P_3 V_3}{T_3} = \frac{P_4 V_4}{T_4}$

$$\therefore T_4 = \frac{P_4 V_4 T_3}{P_4 V_3}$$
$$= \frac{1{,}49}{1} \times 1\,325$$
$$= 1\,974 \text{ K}$$

Find P_5

$$P_4 V_4^{\gamma} = P_5 V_5^{\gamma}$$
$$\therefore P_5 = P_4$$
$$= \left(\frac{V_4}{V_5}\right)^{\gamma}$$
$$= 5\,516 \times \left(\frac{1{,}49}{15}\right)^{1{,}4}$$
$$= 216{,}3 \text{ kPa}$$

Find T_5

$$\frac{P_4 V_4}{T_4} = \frac{P_5 V_5}{T_5}$$
$$\therefore T_5 = \frac{P_5 V_5 T_4}{P_4 V_4}$$
$$= \frac{216{,}3 \times 15}{5\,516 \times 1{,}49} \times 1\,974$$
$$= 780 \text{ K}$$

2. Find the work done per kg/cycle.

$$W = mR\left[(T_4 - T_3) + \left(\frac{T_4 - T_5 - T_2 + T_1}{\gamma - 1}\right)\right]$$
$$= 1 \times 288 \times \left[649 + \left(\frac{1\,974 - 780 - 1\,100 + 373}{1{,}4 - 1}\right)\right]$$
$$= 288\left(649 + \frac{467}{0{,}4}\right)$$
$$= 288\,(649 + 1\,170)$$
$$= 523\,872 \text{ J}$$
$$= 523{,}9 \text{ kJ}$$

3. Find the heat rejected during exhaust stroke (⑤ to ①).

$$Q = m \times C_V(T_5 - T_1)$$
Now $R = C_P - C_V$
$$C_V = C_P - R$$
$$= 1\,000 - 288$$
$$= 712 \text{ J/kg.K}$$

$$Q = 1 \times 712 \times (780 - 373)$$
$$= 712 \times 407$$
$$= 289\,784 \text{ J}$$
$$= 289{,}8 \text{ kJ}$$

C_V is unknown.

Example 1.18

A compression-ignition engine, working on the composite cycle, takes 2/3 of its total heat supply at constant volume, and 1/3 at constant pressure.

Expansion and compression are adiabatic and heat is rejected at constant volume.

- Compression ratio = 13 : 1.
- Maximum pressure = 4,48 MPa.
- Air intake at 101,4 kPa and 15,6 °C
- For air R = 0,293 kJ/kg.K and C_v = 0,712 kJ/kg.K.

Draw the cycle on a P–V diagram to answer the questions.

1. Determine the temperature at each of the five change points of the cycle.
2. Determine the ideal thermal efficiency of the cycle.

Solutions

1. Find the temperatures.

$$R = C_P - C_v$$
$$\therefore C_P = 0{,}712 + 0{,}293$$
$$= 1{,}005 \text{ kJ/kg.K}$$
$$\therefore \gamma = \frac{1{,}005}{0{,}712}$$
$$= 1{,}412$$

$$P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{\gamma}$$
$$= 101{,}4 \times 13^{1{,}412}$$
$$= 3\,792{,}5 \text{ kPa}$$

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$
$$= 288{,}6 \left(\frac{3\,792{,}5}{101{,}4}\right)^{\frac{0{,}412}{1{,}412}}$$
$$= 831 \text{ K}$$

$$\frac{P_2V_2}{T_2} = \frac{P_3V_3}{T_3}$$

$$\therefore T_3 = \frac{V_3P_3T_2}{V_2P_2}$$

$$= \frac{4\,480}{3\,792{,}5} \times 831$$

$$= 981{,}6\ \text{K}$$

Heat added during combustion from ② to ③ and ③ to ④:

$$Q(2\text{ to }3) = \frac{2}{3}Q(\text{total})$$

$$\therefore Q(\text{total}) = \frac{3}{2}Q(2\text{ to }3) \quad \text{①}$$

$$Q(3\text{ to }4) = Q(\text{total})$$

$$\therefore Q(\text{total}) = 3Q(3\text{ to }4) \quad \text{②}$$

① = ②

$$\therefore \frac{3}{2}Q(2-3) = 3Q(3-4)$$

$$\therefore Q(2-3) = 2Q(3-4)$$

$$m \times C_V(T_3 - T_2) = 2 \times m \times C_P(T_4 - T_3)$$

$$0{,}712(981{,}6 - 830) = 2 \times 1{,}005\,(T_4 - 981{,}6)$$

$$107{,}94 = 2{,}01\,T_4 - 1973$$

$$\therefore T_4 = \frac{107{,}94 + 1973}{2{,}01}$$

$$= 1\,035{,}3\ \text{K}$$

2. Find the volume ratio $\frac{V_5}{V_4}$.

At constant pressure ③ to ④: $\frac{P_3V_3}{T_3} = \frac{P_4V_4}{T_4}$

$$\therefore \frac{V_4}{V_3} = \frac{T_4}{T_3}$$

$$= \frac{1\,035{,}3}{981{,}6}$$

$$= 1{,}055:1$$

$$\therefore \frac{V_5}{V_4} = \frac{V_1}{V_4}$$

$$= \frac{V_1}{V_2} \times \frac{V_3}{V_4}$$

$$= \frac{13}{1} \times \frac{1}{1{,}055}$$

$$= 12{,}32:1$$

$$T_5 = T_4\left(\frac{V_4}{V_5}\right)^{\gamma-1}$$
$$= 1\,035{,}3\left(\frac{1}{12{,}32}\right)^{0{,}412}$$
$$= 367{,}9\ \text{K}$$

3.

$$\text{Thermal } \eta = 1 - \left(\frac{T_5 - T_1}{(T_3 - T_2) + \gamma(T_4 - T_3)}\right)$$
$$= 1 - \left(\frac{(367{,}9 - 288{,}6)}{(981{,}6 - 831) + 1{,}412\ (1\,035{,}3 - 981{,}6)}\right)$$
$$= 1 - \left(\frac{79{,}3}{150{,}6 + 75{,}82}\right)$$
$$= 65\%$$

Exercise 1.3

1. An engine works on the ideal dual-cycle principle. Compression ratio = 12 : 1
Pressure at start of compression = 93 kPa and temperature = 94 °C
The maximum pressure in the cycle is 5 516 kPa, and heat added during constant pressure continues for 1/30 of the stroke. Assume adiabatic compression and expansion; C_p = 1 kJ/kg.K; C_v = 0,714 kJ/kg.K.

Calculate the following:

1.1 pressure after polytropic compression and expansion
1.2 temperature at all main points
1.3 work done per kg
1.4 heat supplied per kg

2. A single-cylinder engine working on the dual-cycle principle has a cylinder diameter of 78,5 mm and a stroke of 93,1 mm. The clearance volume is 1/11 of the swept volume. Pressure at start of compression = 69 kPa, with a temperature of 40 °C. Compression and expansion follow the law $PV^{1,33} = C$; R = 0,288 kJ/kg.K; C_p = 1,006 kJ/kg.K and C_v = 0,718 kJ/kg. K. Heat added during constant volume combustion is 0,19 kJ. Pressure at start of exhaust = 330 kPa.

Calculate the following:

2.1 swept volume in cm^3
2.2 clearance and cylinder volume in cm^3
2.3 temperatures after compression and constant volume heat supply
2.4 pressures after compression and constant volume heat supply
2.5 temperatures after and before polytropic expansion
2.6 volume–compression ratio, and fuel cut-off ratio
2.7 heat supplied during combustion

3. The data below refers to a single-cylinder engine working on the dual-cycle principle. Stroke volume = 6 liters. Clearance volume = 1/15 of the stroke volume; Fuel cut-off ratio = 3:1; P_1 = 69 kPa; P_5 = 310,5 kPa; T_1 = 60 °C; C_v = 716 J/kg.K; R = 287 J/kg.K; index for compression and expansion = 1,3.

Calculate the following:
3.1 volumes at all the main points in liters
3.2 temperatures at all the main points
3.3 pressures before and after constant volume heat addition
3.4 work done
3.5..heat supplied
3.6 mean effective pressure

4. An engine working on the dual-cycle principle has a swept volume of 16 400 cm^3 and a clearance volume of 1 024 cm^3. Compression and expansion follow the law $PV^{1,35}$ = C. Highest pressure in the cycle is 1,193 times the pressure at end of compression stroke. Heat added during constant pressure combustion = heat added during constant volume combustion. Pressure and temperature at beginning of compression stroke are 104 kPa and 375 K respectively. C_p = 1 005 J/Kg.K and C_v = 718 J/kg.K.

Calculate the following:
4.1 cylinder volume in cm^3
4.2 pressure and temperature at end of compression
4.3 pressure and temperature at end of constant volume heat addition
4.4 total heat supply
4.5 temperature and volume after constant pressure heat addition
4.6 temperature and pressure before exhaust
4.7 heat rejected

The Joule or constant pressure cycle

The Joule or constant pressure cycle is used on modern turbine engines. The pressure–volume and temperature–entropy diagrams below represent this cycle.

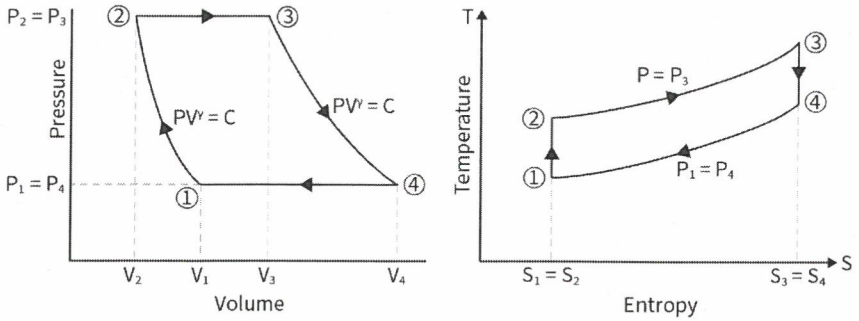


Figure 1.8

Arrangement of the cycle

1 to 2

Adiabatic compression according to law $PV^{\gamma} = C$. Pressure increases from P_1 to P_2 Temperature increases from T_1 to T_2 Volume decreases from V_1 to V_2 Entropy remains constant at $S_1 = S_2$

2 to 3

Heat is added at constant pressure. Pressure remains constant at $P_2 = P_3$ Temperature increases from T_2 to T_3 Volume increases from V_2 to V_3 Entropy increases from S_2 to S_3

3 to 4

Adiabatic expansion according to law $PV^{\gamma} = C$. Pressure decreases from P_3 to P_4 Temperature decreases from T_3 to T_4 Volume increases from V_3 to V_4 Entropy remains constant at $S_3 = S_4$

4 to 1

Heat is rejected at constant pressure. Pressure remains constant at $P_4 = P_1$ Temperature decreases from T_4 to T_1 Volume decreases from V_4 to V_1 Entropy decreases from S_4 to S_1

This process completes the ideal or adiabatic cycle, and the gas return to its original state.

The ideal and actual cycles of an engine are superimposed on the same P–V diagram below.

2' and 4' refer to the ideal cycle with adiabatic processes.\

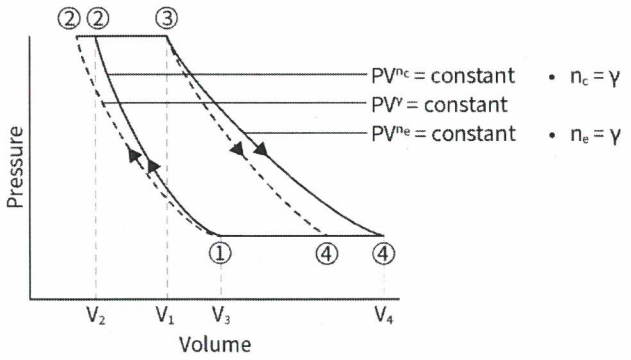


Figure 1.9

The actual work (polytropic) done per cycle in the diagram:

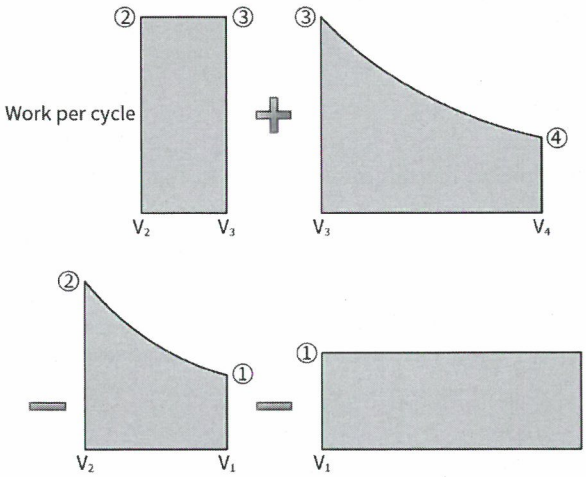


Figure 1.10

Given: n_e = Index for expansion
n_c=Index for compression
m = *mass* of air in kilogram
R = *gas* constant in kJ/kg.K

$$W = (\text{area } V_2, 2, 3, V_3) + (\text{area } V_3, 3, 4, V_4) - (\text{area } V_2, 2, 1, V_1) - (\text{area } V_1, 1, 4, V_4)$$
$$= P_2(V_3 - V_2) + \left(\frac{P_3V_3 - P_4V_4}{n_e - 1}\right) - \left(\frac{P_2V_2 - P_1V_1}{n_c - 1}\right) - P_1(V_4 - V_1)$$
$$= (P_3V_3 - P_2V_2) + \left(\frac{P_3V_3 - P_4V_4}{n_e - 1}\right) - \left(\frac{P_2V_2 - P_1V_1}{n_c - 1}\right) - (P_4V_4 - P_1V_1)$$

$$= mR(T_3 - T_2) + \left(\frac{mR(T_3 - T_4)}{n_e - 1}\right) - \left(\frac{mR(T_2 - T_1)}{n_c - 1}\right) - mR(T_4 - T_1)$$

$$W = mR\left[(T_3 - T_2) + \left(\frac{T_3 - T_4}{n_e - 1}\right) - \left(\frac{T_2 - T_1}{n_c - 1}\right) - (T_4 - T_1)\right] \text{ kJ/cycle}$$

If the index for compression and expansion are the same value:

$$W/\text{cycle} = mR\left[(T_3 - T_2) + \left(\frac{T_3 - T_4 - T_2 + T_1}{n-1}\right) - (T_4 - T_1)\right] \text{ kJ/cycle}$$

The ideal or adiabatic work done per cycle:

$$W/\text{cycle} = mR\left[(T_3 - T_2') + \left(\frac{T_3 - T_4' - T_2' + T_1}{\gamma - 1}\right) - (T_4' - T_1)\right] \text{ kJ/cycle,}$$

where T_4' and T_2' are adiabatic temperatures.

The air standard efficiency of the Joule cycle

Heat added at constant pressure (2 to 3) = $m \times C_P \times (T_3 - T_2)$ joules.

Heat rejected at constant pressure (4 to 1) = $m \times C_P \times (T_4 - T_1)$ joules.

$$\eta_a = \frac{\text{heat received} - \text{heat rejected}}{\text{heat received}}$$

$$= 1 - \frac{\text{heat rejected}}{\text{heat received}}$$

$$= 1 - \frac{mC_P(T_4 - T_1)}{mC_P(T_3 - T_2)}$$

$$\eta_a = 1 - \frac{T_4 - T_1}{T_3 - T_2} \quad \ldots\ldots\ldots\

this thermal energy, which is supplied to the compressor through a shaft from the turbine: the air is receiving heat
while it is being compressed. This is a polytropic process, with the polytropic index $n > \gamma$. The compression is super-adiabatic.

Similarly, in the turbine the air is heated due to friction during an expansion. This is also a polytropic process, with the polytropic index $n < \gamma$. The expansion is sub-adiabatic.

Note

- Gaining heat during compression is the inverse of losing heat during expansion. Both these processes are super-adiabatic.
- Losing heat during compression is the inverse of gaining heat during expansion. Both processes are sub-adiabatic.

The continuous-combustion constant-pressure turbine

The working principle of the continuous-combustion constant pressure turbine is based on the Joule cycle. The turbine consists of three elements: an air compressor, a combustion chamber and a turbine.

Air enters the air compressor, where it is compressed through a pressure–compression ratio of 4 : 1, and sometimes as high as 10 : 1 or even 18 : 1. The air compressors are usually of the rotary type and are either radial or axial flow. The axial flow compressor is illustrated in the figure below.

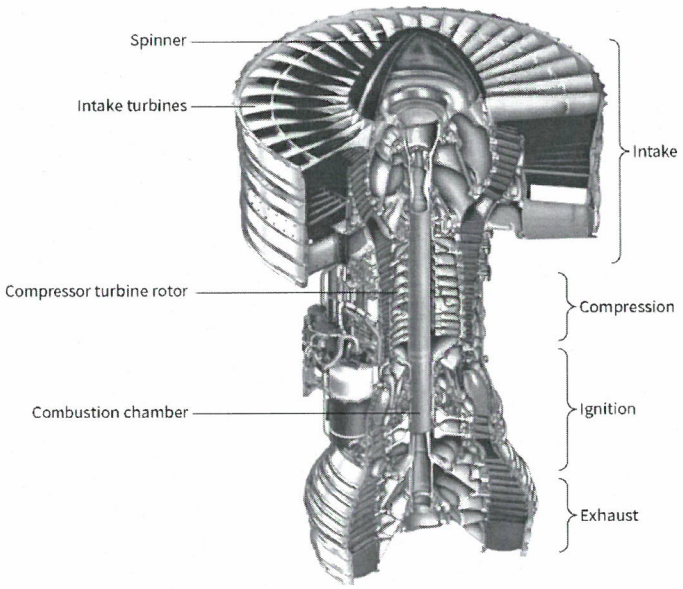


Figure 1.11

$$W = mR\left[(T_3 - T_2) + \left(\frac{T_3 - T_4' - T_2' + T_1}{\gamma - 1}\right) - (T_4' - T_1)\right] \text{ kJ/cycle}$$

Isentropic work done = heat added – heat rejected

$W_i = m \times C_p(T_3 - T_2') - m \times C_p(T_4' - T_1')$

$P_i = \dot{m} \times C_p(T_3 - T_2' - T_4' + T_1)$

This is the frictionless adiabatic or isentropic power.

$$\eta_a = \frac{T_3 - T_4 - T_2 + T_1}{T_3 - T_2}$$

Definitions

Compressor adiabatic efficiency – the ratio of adiabatic temperature rise during compression to actual temperature rise during compression

Compressor adiabatic $\eta_c = \frac{T_{2'} - T_1}{T_2 - T_1}$

Turbine adiabatic efficiency – the ratio of actual temperature drop during expansion to adiabatic temperature drop during expansion

Turbine adiabatic $\eta_t = \frac{T_3 - T_4}{T_3 - T_{4'}}$

Example 1.19

Calculate the air standard efficiency for a gas turbine engine working on the ideal Joule cycle, if the pressure ratio = 6 : 1 and gamma = 1,4 for air.

Solutions

$$\eta_a = 1 - \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \left(\frac{1}{6}\right)^{\frac{1,4-1}{1,4}} = 0,4 = 40\%$$

Example 1.20

A gas turbine engine works on a cycle where air is compressed adiabatically from 101,4 kPa and 20 °C to 275,8 kPa. Heat is added at constant pressure and the hot gas expands adiabatically to atmospheric pressure of 101,4 kPa; gamma = 1,4 and the gas constant = 0,289 kJ/kg.K. Maximum temperature in the cycle = 550 °C.

1. Calculate the temperatures T_2 and T_4.
2. Calculate the heat discharged (Q) in the exhaust per kg of air.
3. Calculate the air standard efficiency of the cycle.

Solutions

1.

Find T_2'

$$T_2' = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 293\left(\frac{275,8}{101,4}\right)^{\frac{0,4}{1,4}} = 390\ K$$

Find T_4'

$$T_4' = T_3\left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = 823\left(\frac{101,4}{275,8}\right)^{\frac{0,4}{1,4}} = 618\ K$$

2.

$Q = m \times C_p(T_4 - T_1)$ joules; C_p is unknown

But $\gamma = \frac{C_p}{C_v}$

$\therefore 1,4 = \frac{C_p}{C_v}$

$C_p = 1,4\ C_v$

From $R = C_p - C_v$

$= 0,289 = 1,4\ C_v - C_v$

$0,289 = 0,4\ C_v$

$C_v = 0,723$ kJ/kg.K

$C_p = 1,4\ C_v = 1,4 \times 0,723 = 1,0115$ kJ/kgK

$Q = 1 \times 1,0115 \times (618 - 293) = 328,738$ kJ

3. $\eta_a = 1 - \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \left(\frac{101,4}{275,8}\right)^{\frac{0,4}{1,4}} = 25\%$

Example 1.21

A gas turbine engine working on the constant pressure cycle has a compression adiabatic efficiency
of 85%. The ambient air pressure and temperature are 103,4 kPa and 288 K respectively; the actual air temperature after compression is 515 K; gamma = 1,4; C_p = 1 kJ/kg.K; 643 kJ of heat is added per kilogram of air per cycle during combustion. Expansion adiabatic efficiency = 82%.

1. Calculate the actual and adiabatic temperatures in Kelvin.
2. Calculate highest pressure in cycle in kPa.
3. Calculate the compression and expansion indices.

4. Calculate the heat lost during constant pressure heat rejection per kilogram of air per cycle.

Solutions

1.

Find T_2'.

Compression adiabatic $\eta = \frac{T_2' - T_1}{T_2 - T_1}$

$0{,}85 = \frac{T_2' - 288}{515 - 288}$

$T_2' = 481\ K$

Find T_3

$Q = m \times C_P \times (T_3 - T_2)$

$643 = 1 \times 1 \times (T_3 - 515)$

$T_3 = 643 + 515$

$= 1\ 158\ K$

To find T_4', first find P_2

$\frac{T_2'}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$

$\therefore P_2 = P_1 \left(\frac{T_2'}{T_1}\right)^{\frac{\gamma-1}{\gamma}}$

$= 103{,}4 \left(\frac{481}{288}\right)^{\frac{1{,}4}{0{,}4}}$

$= 620{,}4\ kPa$

$T_4' = T_3 \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}}$

$= 1\ 158 \left(\frac{103{,}4}{620{,}4}\right)^{\frac{1{,}4-1}{1{,}4}}$

$= 692\ K$

Find T_4

Expansion adiabatic $\eta = \frac{T_3 - T_4}{T_3 - T_4'}$

$0{,}82 = \frac{1\ 158 - T_4}{1\ 158 - 692}$

$T_4 = 776\ K$

2.

Highest press in cycle $= P_2$

$= P_3$

$= 620{,}4$ kPa (already calculated)

3. Find the compression and expansion indices (n_c and n_e)

Compression

$$\frac{T_2}{T_1} = \left(\frac{P}{P_1}\right)^{\frac{n_c-1}{n_c}}$$

$$\frac{515}{288} = \frac{620{,}4}{103{,}4}^{\frac{n_c-1}{n_c}}$$

$$1{,}79 = 6^{\frac{n_c-1}{n_c}}$$

$$1{,}79^{\frac{n_c-1}{n_c}} = 6$$

$$\frac{n_c-1}{n_c} = 3{,}08$$

$$n_c = 1{,}48$$

Expansion

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{n_c-1}{n_c}}$$

$$\therefore \frac{1\,158}{776} = \left(\frac{620{,}4}{103{,}4}\right)^{\frac{n_c-1}{n_c}}$$

$$1{,}49 = (6)^{\frac{n_c-1}{n_c}}$$

$$(1{,}49)^{\frac{n_c-1}{n_c}} = 6$$

$$\therefore \frac{n_e}{n_e-1} = 4{,}5$$

$$n_e = 1{,}287$$

4. Find the heat lost (Q).

$$Q = m \times C_p(T_4 - T_1) = 1 \times 1 \times (776 - 288) = 488 \text{ kJ}$$

Example 1.22

In a continuous-combustion gas turbine, air is taken in at a pressure of 100 kPa and a temperature of 18 °C. It is compressed through a pressure ratio of 5 : 1 with an isentropic efficiency of 85%. In the combustion chamber the temperature is raised to 810 °C. From the combustion chamber the air is passed to a gas turbine, where it is expanded down to 100

kPa with an isentropic efficiency of 88%. From the turbine the air is passed to exhaust. Air used = 4,5 kg/s and neglect the mass of fuel. Assume C_p = 1,006 kJ/kg.K and gamma = 1,4.

1. Determine the net power output of the turbine if the turbine is coupled to the compressor.
2. Determine the air standard efficiency of the plant.

Solutions

1. Find the net power output (for compressor).

$$T_1 = 18 + 273 = 291\ K$$

$$T_2' = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 291\left(\frac{5}{1}\right)^{\frac{0,4}{1,4}}$$

$$= 1\,083\ K$$

$$T_3 = 810 + 273 = 461\ K$$

$$\eta_c = \frac{T_2'-T_1}{T_2-T_1}$$

$$0,85 = \frac{461-291}{T_2-291}$$

$$T_2 = 491\ K$$

For the turbine:

$$T_2' = T_3\left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 1\,083\left(\frac{1}{5}\right)^{\frac{0,4}{1,4}}$$

$$= 683\ K$$

$$\eta_t = \frac{T_3-T_4}{T_3-T_4'}$$

$$0,88 = \frac{1\,083-T_4}{1\,083-683}$$

$$T_4 = 731\ K$$

$$P_n = mC_P(T_3 - T_4 - T_2 + T_1)$$

$$= 4,5 \times 1,006(1\,083 - 731 - 491 + 291)$$

$$P_n = 688\ kW$$

2.

$$\eta_a = \frac{T_3 - T_4 - T_2 + T_1}{T_3 - T_2}$$

$$= \frac{1\,083 - 731 - 491 + 291}{T_3 - T_2}$$

$$= \frac{152}{592}$$

$$\eta_a = 25{,}7\%$$

Exercise 1.4

1. A gas turbine plant operates on the ideal Joule cycle. The air enters the compressor at 100 kPa and 15 °C and is delivered at 600 kPa. The temperature at the turbine inlet = 650 °C. Assume gamma = 1,4; C_p = 1,005 kJ/kgK and R = 0,287 kJ/kgK.

Calculate the following:

1.1 heat supplied per kg

1.2 compressor work done per kg

1.3 turbine work done per kg

1.4 heat rejected per kg

1.5 air standard efficiency of the plant

2. A gas engine turbine working on the Joule cycle has a compression adiabatic efficiency of 83% and an expansion adiabatic efficiency of 80%.
Adiabatic index = 1,4; P_1 = 103,4 kPa; T_1 = 15 °C;T_2 = 237 °C; C_p = 1 kJ/kgK.
600 kJ of heat is added per kg of air during constant pressure combustion.

Calculate the following:

2.1 all adiabatic and actual temperatures

2.2 highest pressure in the cycle

2.3 compression and expansion indices

3. The data below refers to a gas turbine engine working on the Joule cycle.
P_1 = 103 kPa; T_1 = 293 K; P_2 = 824 kPa. Compression adiabatic efficiency = 84% Expansion adiabatic efficiency = 82%. Gamma for air = 1,4; C_p = 1,005 kJ/kgK
Heat added during combustion = 530 kJ per kg of air; mass flow = 50 kg/s;
Heat value of fuel = 46 MJ/kg

Calculate the following:

3.1 all adiabatic and actual temperatures

3.2 mass of fuel used per second

3.3 air–fuel ratio

3.4 compression and expansion indices.

The spark ignition engine (petrol)
When the mixture of fuel and air is ignited inside the cylinder of the engine by a spark plug, it is called a spark ignition engine and can operate on the four- or two-stroke cycle principle.

The compression ignition (diesel) engine
The compression ignition engine does not use an ignition system and spark plugs to ignite the mixture of fuel and air in the cylinder towards the end of the compression stroke.

The compression ratio of the cylinder is rather high, resulting in compressed air at a very high temperature toward the end of the compression stroke.

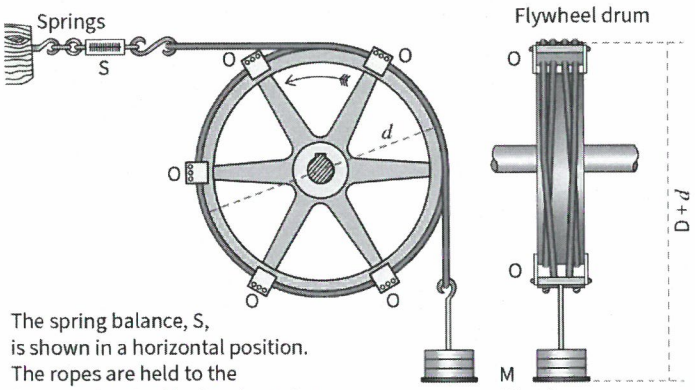
Just before top dead centre (TDC), the fuel is injected into the compressed air and the fuel ignited instantaneously due to the high temperature. The fuel is injected into the cylinder by a fuel injector pump, timed to come into operation at a few degrees before TDC on compression stroke. The compression ignition engine can operate on the four-stroke or two-stroke cycle principle, and is usually a diesel engine.

Measuring the brake power (BP) of an IC engine
The rope brake

1.1.2 Calculations on IC engine power and efficiencies

Formula
$P_b = 2\pi N(W - S)R_e$

Where:
W = dead load (N)
S = spring balance value(N)
R_e = effective radius (m)
N = rotational frequency (rps)



The spring balance, S, is shown in a horizontal position. The ropes are held to the flywheel face by blocks of wood, O.

Figure 1.12

The rope is wound once around the drum, which is attached to the wheels or shaft of the engine. The one end of the rope carries a dead load, while the other end is attached to a spring balance.

The direction of the rotation of the flywheel (drum) is opposite to the pull of the dead load. The engine is started without the dead load being applied, and the masses are gradually applied until a reading is registered on the spring scale. As soon as the engine is running steadily at full throttle, the speed is taken with a stopwatch and a rev counter. The brake power can then be calculated with the formula above, which is used only for a rope brake.

Example 1.23

A diesel engine has a brake torque of 269,2 Nm at 180 r/min.

Calculate the brake power of the engine.

Solution

$P_b = 2\pi NT = 2\pi \times 3 \times 269{,}2 \times 10^3 = 5{,}074\ MW$

Example 1.24

A rope brake is used to measure the brake power of an engine gave the following results: diameter of rope = 20 mm; diameter of brake drum = 1,5 m; engine speed = 300 r/min.; spring balance reading = 5,6 kg; mass on end of rope = 40 kg.

Calculate the brake power of the engine.

Solutions

$W = 40 \times 9{,}81 = 392{,}4\ N$

$S = 5{,}6 \times 9{,}81 = 54{,}936\ N$

$$R_e = \frac{D + d}{2}$$
$$= \frac{1{,}5 + 0{,}02}{2} = 0{,}76 \text{ m}$$

$$P_b = 2\pi N(W - S)R_e$$
$$= 2\pi \times 5(392{,}4 - 54{,}936)0{,}76$$
$$= 8{,}057 \text{ kW}$$

Example 1.25

An engine was tested with a rope brake, with these results:
brake power = 9 kW; diameter of brake drum = 1,6 m; diameter of rope = 22 mm; mass at end of rope = 42 kg; spring balance reading = 6 kg.

Calculate the rotational speed of the engine

Solutions

$$N = \frac{P_b}{2\pi(W - S)R_e}$$
$$= \frac{9 \times 10^3}{2\pi(353{,}16)0{,}811}$$
$$= 5 \text{ rps} = 300 \text{ rpm}$$

Prony brake

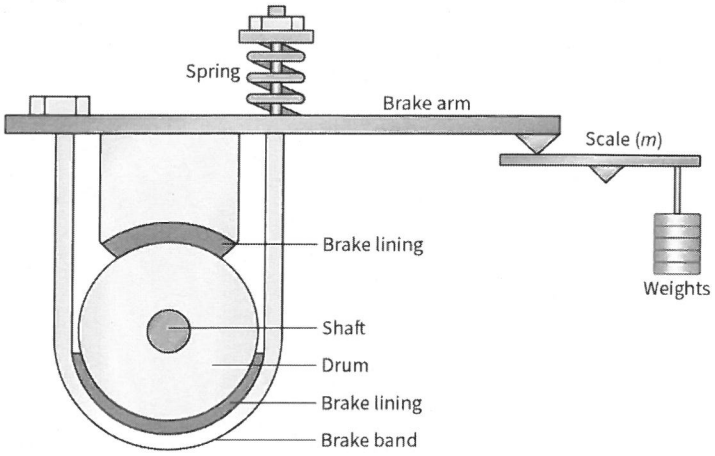


Figure 1.13

Formula

$$P_b = 2\pi NT$$

Where:

T is torque (Nm)

$T = W \times L$

$W = mg$

N is rotational frequency (rpm)

Example 1.26

During a brake power test with a Prony brake, these results were obtained: mass on end of lever arm = 47,6 kg; length of lever arm = 914 mm; engine speed = 120 rpm.

Calculate the brake power.

Solution

$T = W \times L$

$= 466{,}956 \times 0{,}914 = 426{,}798$ Nm

$P_b = 2\pi NT$

$= 2\pi \times 2 \times 426{,}798 = 5{,}363$ kW

Example 1.30

An engine was tested with a Prony brake. The results were brake power = 656 kW and brake torque = 3,132 kNm. The length of the lever arm is 1,6 m.

Calculate the following:

1. rotational speed
2. balance mass at the end of the lever arm

Solutions

1.

$$N = \frac{P_b}{2\pi T} = \frac{656 \times 10^3}{2\pi \times 3\,132}$$

$= 33{,}335$ rps $= 2000{,}108$ rpm

2.

$$m = \frac{T}{gL}$$

$$= \frac{3\,132}{9{,}81 \times 1{,}6} = 199{,}541 \text{ kg}$$

Indicated power by Morse test

Due to the mechanical nature of the engine indicator, it is unsuitable for use on high-speed engines. The inertia of its moving parts, friction and the possibility of

control-spring vibration restrict its use to speeds of under 960 r/min. This makes its use for determining the indicated power of high-speed engines impossible, so other methods must be used.

One method to obtain a close estimate of the indicated power of a multi-cylinder internal combustion engine is the Morse test. This method can only be used on engines with more than one cylinder.

The engine being tested is coupled to a suitable brake and, at the test condition, the brake power output is determined. Let this be BP.

The first cylinder is now cut out. For a petrol engine, this can be done by shorting out the spark plug of the first cylinder. For an oil engine, the fuel supply to the first cylinder is interrupted. As a result of cutting out the first cylinder, the engine speed will drop. Load is now removed from the brake in order to restore the original speed. The brake power under this new condition is determined from the new brake load. The first cylinder operation is restored and the second cylinder is cut out. If necessary, the engine speed is corrected to its original value, and the brake power with the second cylinder cut-out is determined.

This procedure is applied to each cylinder. A four-cylinder engine is used for the example below.
Let I_1, I_2, I_3, I_4 be the indicated power of each cylinder.
Let F_1, F_2, F_3, F_4 be the friction power of each cylinder.

When all cylinders are working normally:
Total: $BP = (I_1 - F_1) + (I_2 - F_2) + (I_3 - F_3) + (I_4 - F_4)$
Brake power $= (I_1 + I_2 + I_3 + I_4) - (F_1 + F_2 + F_3 + F_4)$ 1

When the first cylinder is cut out, I_1 is cut out.
The friction and other losses of the first cylinder remain.

So, the brake power with the first cylinder cut out:
$BP_1 = (I_2 + I_3 + I_4) - (F_1 + F_2 + F_3 + F_4)$ 2
Subtract 2 from 1: $BP - BP_1 = I_1$ 3
The indicated power of the first cylinder is determined in a similar way when the second cylinder is cut out.
$BP - BP_2 = I_2$

This is repeated for each cylinder in turn.
Finally, the total indicated power: $IP = I_1 + I_2 + I_3 + I_4$

The indicated power determined in this way is not totally accurate, as the friction and pumping losses when a cylinder is not working are not the same as when it is working. However, the method gives a close approximation.

Example 1.27

In a trial on a six-cylinder petrol engine, a Morse test was carried out to estimate the indicated power of the engine. Running at full load, with all cylinders working, the brake power output was 52 kW. The measured brake power output (in kW) when each cylinder was cut out in turn, and the load reduced to bring the engine back to its original speed, are shown in the table.

Cylinder	1	2	3	4	5	6
Brake power (kW)	40,5	40,2	40,1	40,6	40,7	40

Use the data in the table to estimate the indicated power of the engine

Solution

$I_1 = 52 - 40{,}5 = 11{,}5$ kW
$I_2 = 52 - 40{,}2 = 11{,}8$ kW
$I_1 = 52 - 40{,}1 = 11{,}9$ kW
$I_1 = 52 - 40{,}6 = 11{,}4$ kW
$I_1 = 52 - 40{,}7 = 11{,}3$ kW
$I_1 = 52 - 40{,}0 = 12{,}0$ kW

$I_T = P_i = 69{,}9$ kW

Exercise 1.5

1. A rope of 17 mm diameter is wound once around drum of 1 m diameter. The one end of the rope is tied to a spring balance, and the other end to a mass of 50 kg. Engine speed is 500 r/min. and the spring balance reading = 30 N.

Calculate the brake power.

2. In a rope brake test, the diameter of the drum = 0,6 m and the diameter of the rope = 16 mm. The effective load is 3 520 kg, and the engine speed is 210 r/min.

Calculate the brake power.

3. These are the results of an engine tested with a rope brake:
brake power = 21 kW; rope diameter = 30 mm; brake diameter = 1,2 m;
mass on end of rope = 50 kg; spring balance reading = 7 kg.

Calculate the speed of the engine.

4. Calculate the brake power of an engine tested with a Prony brake at 600 r/min.
The brake arm length is 1 100 mm and the mass on the end of the lever arm is 47 kg.

5. An engine tested with a Prony brake gave these results: brake power = 328 kW; mass on end of arm = 88 kg; length of lever arm = 1,8 m.

Calculate the speed of the engine.

Indicated power (P_i)

The energy liberated during the combustion of fuel inside the cylinders of an IC engine is called the indicated power. It can be calculated with the formula below.

$$P_i = p_{im}LANEx$$

Where:
P_i is indicated power (W)
p_{im} is indicated mean effective pressure (Pa)
L is stroke length (m)
N is rotational frequency (rps)

E = 0,5 for four stroke engine
E = 1 for two stroke engine
E = 2 for double acting
x is number of cylinders

Example 1.28

A 6-cylinder, two-stroke diesel engine has a cylinder diameter of 456 mm and a stroke of 600 mm. The engine speed is 180 rpm. The indicated mean effective pressure is 1,242 MPa. Calculate the indicated power

$$A = \frac{\pi D^2}{4}$$
$$= \frac{\pi \times 0{,}457^2}{4} = 0{,}164\ m^2$$

$$P_i = p_{im}LANEx$$
$$= 1{,}242 \times 10^6 \times 0{,}6 \times 0{,}164 \times 3 \times 1 \times 6$$
$$= 2{,}2\ MW$$

Example 1.29

A six-cylinder, four-stroke petrol engine gave these results in a test: cylinder diameter = 240 mm; stroke = 305 mm; torque = 3,1 kNm at 1 800 r/min. The indicated power developed 800 kW.

Calculate the following:

1. brake power
2. indicated mean effective pressure

Solutions

1.

$$P_b = 2\pi NT$$
$$= 2\pi \times 30 \times 3\,100$$
$$= 584{,}336 \text{ kW}$$

$$A = \frac{\pi D^2}{4}$$
$$= \frac{\pi \times 0{,}24^2}{4} = 0{,}045 \text{ m}^2$$

2.

$$p_{im} = \frac{P_i}{LANEx}$$
$$= \frac{800 \times 10^3}{0{,}305 \times 0{,}045 \times 30 \times 0{,}5 \times 6}$$
$$p_{im} = 644{,}222 \text{ kPa}$$

Example 1.30

Calculate the cylinder diameter and stroke of a single-cylinder, four-stroke petrol engine that runs at 1 500 rpm and develops an indicated power of 30 kW. The indicated mean effective pressure is 368 kPa. The stroke length is 1,5 times the cylinder diameter.

Solution

$$P_i = p_{im}LANEx$$
$$30 \times 10^3 = 368 \times 10^3 \times 1{,}5D \times 0{,}785 \times D^2 \times 25 \times 0{,}5 \times 1$$
$$D = 176{,}9 \text{ mm}$$

Mechanical efficiency (η_m)

The ratio of the indicated power to the brake power is known as the mechanical efficiency.

$$\eta_m = \frac{P_b}{P_i}$$

Example 1.31

These results were obtained from a test on a single-cylinder, four-stroke diesel engine running at 200 rpm.

- mean effective pressure is 602 kPa
- cylinder diameter is 305 mm
- stroke length is 460 mm
- brake load at 1,4 m is 85 kg

Calculate the following:

1. the brake power
2. the indicated power
3. the mechanical efficiency

Solutions

1.

$$T = WL = 85 \times 9{,}81 \times 1{,}4$$
$$= 1{,}167 \text{ kNm}$$

$$P_b = 2\pi NT$$
$$= 2\pi \times 3{,}333 \times 1\,167$$
$$= 24{,}45 \text{ kW}$$

2.

$$A = \frac{\pi D^2}{4}$$
$$= \frac{\pi \times 0{,}305^2}{4}$$
$$= 0{,}073 \text{ m}^2$$

$$P_i = p_{im} LANEx$$
$$= 602 \times 10^3 \times 0{,}46 \times 0{,}073 \times 3{,}33 \times 0{,}5 \times 1$$
$$= 33{,}72 \text{ kW}$$

3.

$$\eta_m = \frac{P_b}{P_i}$$
$$= \frac{24{,}45}{33{,}72} = 0{,}725 = 72{,}507\%$$

Example 1.32

A six-cylinder, four-stroke petrol engine gave these results: piston diameter is 240 mm; stroke is 305 mm; mechanical efficiency is 80%; brake torque is 3 kNm at 210 r/min.

Calculate the following:

1. The brake power
2. The indicated power

3. The indicated mean effective pressure

Solutions

1.

$$P_b = 2\pi NT$$
$$= 2\pi \times 3{,}5 \times 3 \times 10^3$$
$$= 65{,}973 \text{ kW}$$

2.

$$P_i = \frac{P_b}{\eta_m}$$
$$= \frac{65{,}973}{0{,}8} = 0{,}725$$
$$= 82{,}467 \text{ kW}$$

3.

$$A = \frac{\pi D^2}{4}$$
$$= \frac{\pi \times 0{,}24^2}{4} = 0{,}045 \text{ m}^2$$

$$p_{im} = \frac{P_i}{LANEx}$$
$$= \frac{82{,}467 \times 10^3}{0{,}305 \times 0{,}045 \times 3{,}5 \times 0{,}5 \times 6}$$
$$p_{im} = 569{,}217 \text{ kPa}$$

Example 1.33

An indicator diagram obtained from a test on an engine, has a length of 70 mm and an area of 19,6 cm^2. The spring rate of the indicator is 20 kPa/mm.

Calculate the indicated mean effective pressure.

Solution

$$p_{im} = \frac{A}{\ell} \times sr$$
$$= \frac{19{,}6 \times 10^2}{70} \times 20 = 560 \text{ kPa}$$

The indicated mean effective pressure of an IC engine

The indicated mean effective pressure (IMEP) of an IC engine is measured with an engine indicator.

In internal combustion engines and steam engines work on the piston is done by a working agent under pressure. This pressure does not stay constant inside the cylinder.

To find the amount of work done on the cylinder of such an engine, it is essential to plot a pressure–volume diagram during the movement of the piston inside the cylinder. This is done automatically by the engine indicator, as shown below.

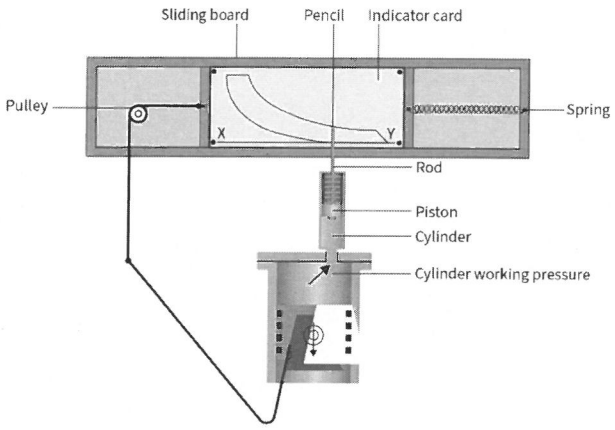


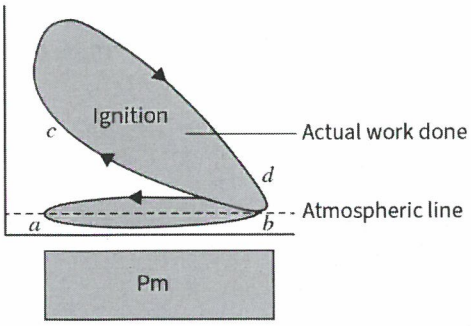
Figure 1.14

A small cylinder can be connected to the engine cylinder by means of a valve, so that the pressure in the engine cylinder acts on a small piston, tending to move it upwards. This movement is controlled by a spring, so the upward motion will be directly proportional to the force on the piston. This motion is magnified by a linkage to give an upward movement of the tracing point.

The diagram is drawn on a sheet of paper with a surface coating, which is marked by the tracing point. The paper is attached to a drum that is rotated by a cord, so that motion is directly proportional to the motion of the engine piston.

The tracing point has a vertical motion that is directly proportional to the pressure in the engine cylinder, and the paper has a horizontal motion that is directly proportional to the movement of the piston, hence the stroke volume (swept volume) of the piston.

If the tracing point is kept in contact with the paper for a complete cycle, the resultant traced diagram will be a pressure–volume diagram, called an indicator diagram, which is illustrated on the next page.



a–b: induction stroke
b–c: compression stroke
c–d: power stroke
d–a: exhaust stroke

Figure 1.15

A typical indicator diagram from a petrol engine operating on the four-stroke cycle is shown in the diagram. The work done in the cylinder of an engine can be determined by finding the area of the indicator diagram with a mid-ordinate rule, a Simpson's rule or a planimeter.

The mean effective pressure (p_m) can be found from the above diagram. The pressure–volume diagram can be represented by a rectangle with height p_m.

p_m is the average (mean) height of the diagram:

p_m = (average height of diagram) × (spring stiffness)

p_m = (height in mm) × (spring stiffness in mm)

Exercise 1.6

1. An indicator diagram obtained from a four-stroke single cylinder engine has a length of 75 mm and an area of 6,24 cm^2. The spring stiffness of the indicator = 100 kPa per mm. The engine has a piston diameter of 250 mm and a stroke of 400 mm. It rotates at 360 r/min.

Calculate the following:

1.1 mean effective pressure.

1.2 indicated power.

1.3 brake power if the mechanical efficiency is 80%.

2. A four-stroke, four-cylinder engine has a piston diameter of 80 mm and a stroke of 75 mm. The mean effective pressure at 2 000 r/min = 700 kPa, with a mechanical efficiency of 82%.

Calculate the following:

2.1 swept volume in cm^3

2.2 indicated power

2.3 brake power

3. This information was obtained from a test on single-cylinder, four-stroke engine at 320 r/min:
area of indicator diagram = 780 mm^2; length of indicator diagram = 75 mm; indicator spring stiffness = 90 kPa per mm; cylinder diameter = 200 mm; stroke length = 350 mm; net brake load = 110 kg; drum radius = 0,6 m.

Calculate the following:
3.1 indicated power
3.2 brake power
3.3 mechanical efficiency

4. A four-stroke, single cylinder engine develops an indicated power of 50 kW at 360 r/min. The indicated mean effective pressure = 820 kPa, the stroke length = 400 mm and the mechanical efficiency is 80%.

Calculate the following:
4.1 brake power
4.2 piston diameter

5. The brake power of a four-cylinder, four-stroke petrol engine = 15 kW. The mechanical efficiency is 82%. The mean affective pressure at 2 000 r/min is 700 kPa and the piston diameter = 80 mm.

Calculate the following:
5.1 indicated power
5.2 length of stroke

6. Information obtained from a test on a six-cylinder, four-stroke engine at 350 r/min.is as follows:
area of indicator diagram = 800 mm^2; length of diagram = 77 mm; indicator spring stiffness = 92 kPa per mm; cylinder diameter = 210 mm; stroke =340 mm; net brake load on Prony brake = 600 kg; length of lever arm = 0,65 m.

Calculate the following:

6.1 indicated mean effective pressure
6.2 indicated power
6.3 brake power
6.4 mechanical efficiency

Summary of internal combustion efficiencies

Fuel power

$P_f = \dot{m}_f \times CV$

Indicated power

$P_i = p_{im} \times LANEx$

Brake power

$P_b = 2\pi NT$

Indicated thermal efficiency

$\eta_{it} = \frac{P_i}{P_f}$

Brake thermal efficiency

$\eta_{bt} = \frac{P_b}{P_f}$

Mechanical efficiency

$\eta_m = \frac{P_b}{P_i} = \frac{\eta_{bt}}{\eta_{it}}$

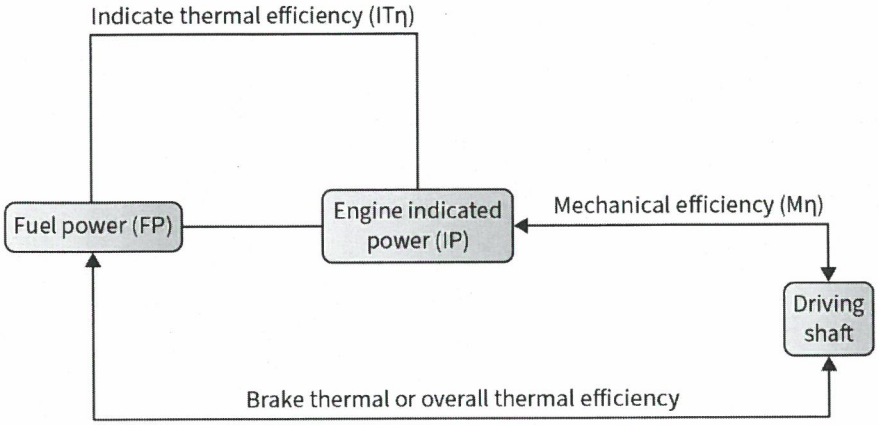


Figure 1.16

1.1.3 Heat balance table for an IC engine

To draw up a heat balance sheet for an ICE, a complete test should be done on the engine running at constant load. Measurements should be taken of all quantities of heat energy supplied or emitted during the test. Fuel consumption should be measured and the calorific value of the fuel must be known. The temperature of the cooling water, the indicated power, the brake power, as well as the analysis and mass of the exhaust gases should be measured. The energy of these factors could then be reduced to a common basis of kilojoules per minute (kJ/min.) and entered on the heat balance sheet.

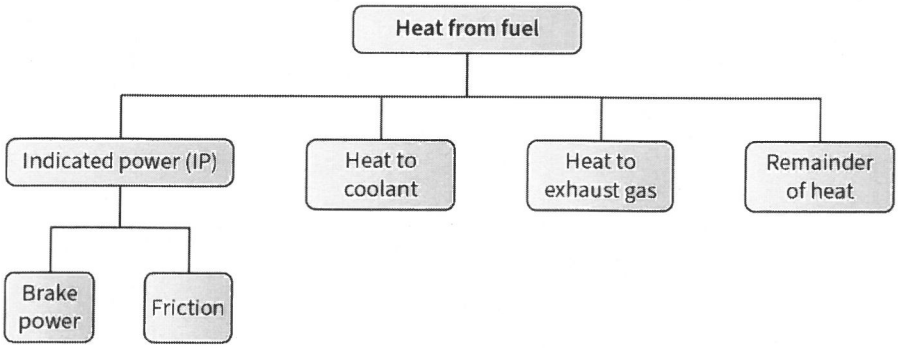


Figure 1.17

Example 1.34

Test data for a double acting two-stroke IC engine:

- number of cylinders = 8
- cylinder diameter = 450 mm
- stroke length = 600 mm
- brake torque = 270 kNm
- rotational speed = 180 rpm
- brake thermal efficiency = 33%
- mechanical efficiency = 85%
- calorific value of fuel = 46,5 MJ/kg
- air/fuel-ratio = 30:1

Calculate the following:

1. brake power
2. indicated power
3. indicated thermal efficiency
4. mass rate of the exhaust gasses
5. indicated mean effective pressure
6. friction power

Solutions

1.

$P_b = 2\pi NT$

$= 2\pi \times 3 \times 270 \times 10^3 = 5{,}089 \text{ MW}$

2.

$$P_i = \frac{P_b}{\eta_m}$$

$$= \frac{5{,}089}{0{,}85} = 5{,}988 \text{ MW}$$

3.

$$\eta_{it} = \frac{\eta_{bt}}{\eta_m}$$

$$= \frac{0,33}{0,85} = 0,388$$
$$= 38,824\%$$

4.

$$P_f = \frac{P_b}{\eta_{bt}}$$
$$= \frac{5,089}{0,33} = 15,422 \text{ MW}$$

$$\dot{m}_f = \frac{P_f}{CV}$$
$$= \frac{15,422}{46,5} = 0,332 \text{ kg/s}$$

$$\dot{m}_g = \dot{m}_f \left(\frac{a}{f} + 1\right)$$
$$= 0,332 \times (30 + 1) = 10,282 \text{ kg/s}$$

5.

$$A = \frac{\pi D^2}{4}$$
$$= \frac{\pi \times 0,45^2}{4} = 0,159 \text{ m}^2$$

$$p_{im} = \frac{P_i}{LANEx}$$
$$= \frac{5,988 \times 10^6}{0,6 \times 0,159 \times 3 \times 2 \times 8}$$
$$p_{im} = 1,307 \text{ MPa}$$

6.

$$P_\mu = P_i - P_b$$
$$= (5,988 - 5,089) \times 10^6 = 898,126 \text{ kW}$$

Example 1.35

Test data for a four-stroke IC engine using indicator diagram and rope brake:

- number of cylinders = 2
- cylinder diameter = 80 mm
- stroke length = 75 mm
- rotational speed = 2 760 rpm
- fuel used = 0,03 kg/min
- calorific value of fuel = 41 MJ/kg
- area of indicator diagram = 7,8 cm^2
- length of indicator diagram = 65 mm
- spring rate = 25 kPa/mm
- diameter of flywheel = 300 mm
- diameter of rope = 15 mm
- mass on rope 11 kg
- spring scale reading = 15 N

Calculate the following:

1. indicated mean effective pressure
2. indicated power
3. brake power
4. brake thermal efficiency
5. mechanical efficiency

Solutions

1.

$$p_{im} = \frac{A}{\ell} \times sr =$$

$$\frac{7{,}8 \times 10^2}{65} \times 25 = 300 \text{ kPa}$$

2.

$$A = \frac{\pi D^2}{4}$$

$$= \frac{\pi \times 0{,}08^2}{4} = 0{,}005 \text{ m}^2$$

$$P_i = p_{im} LANEx$$

$$= 300 \times 10^3 \times 0{,}005 \times 0{,}075 \times 46 \times 0{,}5 \times 2$$

$$P_i = 5{,}202 \text{ kW}$$

3.

$$W = 11 \times 9{,}81 = 107{,}91 \text{ N}$$

$$R_e = \frac{D + d}{2}$$

$$= \frac{0{,}3 + 0{,}015}{2} = 0{,}158 \text{ m}$$

$$P_b = 2\pi N(W - S)R_e$$

$$= 2\pi \times 46(107{,}91 - 15)0{,}158 = 4{,}229 \text{ kW}$$

3.

$$P_f = \dot{m}_f \times CV$$

$$= \frac{0{,}03 \times 41 \times 10^6}{60} = 20{,}5 \text{ kW}$$

$$\eta_{bt} = \frac{P_b}{P_f}$$

$$= \frac{4{,}229}{20{,}5} = 0{,}206$$

$$= 20{,}631\%$$

5.

$$\eta_m = \frac{P_b}{P_i}$$
$$= \frac{4,229}{5,202} = 0,813$$
$$= 81,296\%$$

Example 1.36

Test data for a four stroke IC engine:

- number of cylinders = 1
- cylinder diameter = 190 mm
- stroke length = 360 mm
- load on brake = 400 N
- mean diameter of drum = 1,4 m
- rotational speed = 360 rpm
- mechanical efficiency = 75%
- fuel used = 0,05 kg/min
- calorific value of fuel = 44,5 MJ/kg
- mass rate of cooling water = 5 kg/min
- c_w for water = 4,2 kJ/kgK
- cooling water temperature rise = 25°C
- air/fuel-ratio = 25:1
- C_g for exhaust gas = 1,05 kJ/kgK
- gas temperature rise = 430 °C

Calculate the following:

1. brake power
2. indicated power
3. heat supplied by the fuel per minute
4. heat absorbed by brake power per minute
5. heat lost to friction per minute
6. heat lost to coolant per minute
7. heat lost to exhaust gas per minute
8. percentages of the energy absorbed from the fuel in a heat table

Solutions

1.

$$T = W \times R_e$$
$$= 400 \times 0,7 = 280 \text{ Nm}$$

$$P_b = 2\pi NT$$
$$= 2\pi \times 6 \times 280 = 10,556 \text{ kW}$$

2.

$$P_i = \frac{P_b}{\eta_m}$$

$$= \frac{10{,}556}{0{,}75} = 14{,}074 \text{ kW}$$

3.

$$Q_f = \dot{m}_f \times CV$$
$$= 0{,}05 \times 44{,}5 \times 10^6 = 2{,}225 \text{ MJ/min}$$

4.

$$Q_b = P_b \times 60$$
$$= 10{,}556 \times 60 = 633{,}345 \text{ kJ/min}$$

5.

$$P_\mu = P_i - P_b$$
$$= (14{,}074 - 10{,}556) \times 10^3$$
$$= 3{,}519 \text{ kW}$$

$$Q_\mu = P_\mu \times 60$$
$$= 3{,}519 \times 60$$
$$= 211{,}115 \text{ kJ/min}$$

6.

$$Q_w = \dot{m}_w c_w \Delta t$$
$$= 5 \times 4{,}2 \times 10^3 \times 25$$

$$= 525 \text{ kJ/min}$$

7.

$$\dot{m}_g = \dot{m}_f \left(\frac{a}{f} + 1\right)$$
$$= 0{,}05 \times (25 + 1)$$
$$= 1{,}3 \text{ kg/min}$$

$$Q_g = \dot{m}_g c_g \Delta t$$
$$= 1{,}3 \times 1{,}05 \times 10^3 \times 430$$
$$= 586{,}95 \text{ kJ/min}$$

8.

Energy supply	**Destination**	**Energy absorbed**	**Percentage**
2 225	Brake power	633,345	28,465%
	Coolant	525	23,596%
	Exhaust	586,95	26,38%
	Radiation	484,705	21,56%
Total		**2 225**	**100%**

Example 1.37

A four-cylinder, four-stroke petrol engine is tested on a Prony brake. The brake arm is 1 m long and a mass of 15 kg balances the brake at 2 100 r/min. The mechanical efficiency at this speed is 80% and the brake thermal efficiency is 25%. The air/fuel ratio is 19 : 1. The heat value of the fuel is 44,2 MJ/kg. The specific heat capacity of the exhaust gas is 1,05 kJ/kg.K, with an exhaust gas temperature rise of 570 °C. The heat to the cooling system is the same as the heat to the exhaust gas per minute.

Draw up a heat balance in kJ/min, and express this as a percentage of the heat supplied by the fuel per minute.

Solutions

$T = W \times L =$
$15 \times 9{,}81 \times 1 = 147{,}15$ Nm

$P_b = 2\pi NT$
$= 2\pi \times 35 \times 147{,}15 = 32{,}36$ kW

$Q_b = P_b \times 60$
$= 32{,}36 \times 60 = 1\,941{,}599$ kJ/min

$$P_f = \frac{P_b}{\eta_{bt}} =$$
$$\frac{32{,}36}{0{,}25} = 129{,}44 \text{ kW}$$

$$\dot{m}_f = \frac{P_f}{CV} =$$
$$\frac{129{,}44 \times 10^3 \times 60}{44{,}2 \times 10^6} = 0{,}176 \text{ kg/min}$$

$Q_f = P_f \times 60$
$= 129{,}44 \times 10^3 \times 60$
$= 7\,766{,}394$ kJ/min

$$\dot{m}_g = \dot{m}_f \left(\frac{a}{f} + 1\right)$$
$= 0{,}176 \times (19 + 1)$
$= 3{,}514$ kg/min

$Q_g = \dot{m}_g c_g \Delta t$
$= 3{,}514 \times 1{,}05 \times 10^3 \times 570$
$= 2\,103{,}252$ kJ/min

Energy supply	Destination	Energy absorbed	Percentage

7 766,394	Brake power	1 941,599	25%
	Coolant	2103,252	27,081%
	Exhaust	2103,252	27,081%
	Radiation	1 618,292	20,837%
Total		**2 225**	**100**

Example 1.38

In a trial on a six-cylinder petrol engine, a Morse test was carried out to estimate the indicated power of the engine. Running at full load, with all cylinders working, the brake power output was 56 kW. The measured brake power output (in kW) when each cylinder was cut out in turn, and the load reduced to bring the engine back to its original speed, are shown in the table.

Cylinder	1	2	3	4	5	6
Brake power (kW)	44,2	44	43,9	44,3	44,1	43,7

Further test recordings: fuel consumption = 0,3 kg/min; heat value of fuel = 42 MJ/kg; air/fuel ratio = 15 : 1; exhaust gas temperature = 400 °C; specific heat capacity of exhaust gas = 1,1 kJ/kg.K; outside air temperature = 18 °C; cooling water inlet temperature = 18 °C; cooling water outlet temperature = 56 °C; cooling water flow rate = 34 kg/min. Specific heat capacity for the water is 4,2 kJ/kgK.

Calculate the following:

1. mechanical efficiency
2. heat supplied by the fuel per minute
3. heat absorbed by brake power per minute
4. heat lost to coolant per minute
5. heat lost to exhaust gas per minute
6. percentages of the energy absorbed from the fuel in a heat table

Solutions

1.

$I_1 = 56 - 44{,}2 = 11{,}8\ \text{kW}$
$I_2 = 56 - 44 = 12\ \text{kW}$
$I_1 = 56 - 43{,}9 = 12{,}1\ \text{kW}$
$I_1 = 56 - 44{,}3 = 11{,}7\ \text{kW}$
$I_1 = 56 - 44{,}1 = 11{,}9\ \text{kW}$
$I_1 = 56 - 43{,}7 = 12{,}3\ \text{kW}$

$I_T = P_i = 71{,}8\ \text{kW}$

$$\eta_m = \frac{P_b}{P_i}$$

$= \frac{56}{71,8} = 0,78 =$
77,994%

2.

$Q_f = \dot{m}_f \times CV$
$= 0,3 \times 42 \times 10^6$
$= 12,6$ MJ/min

3.

$Q_b = P_b \times 60$
$= 56 \times 10^3 \times 60$
$= 3,36$ MJ/min

4.

$Q_w = \dot{m}_w c_w \Delta t$
$= 34 \times 4,2 \times 10^3 \times (56 - 18)$
$= 5,426$ MJ/min

5.

$\dot{m}_g = \dot{m}_f \left(\frac{a}{f} + 1\right)$
$= 0,3 \times (15 + 1)$
$= 4,8$ kg/min

$Q_g = \dot{m}_g c_g \Delta t$
$= 4,8 \times 1,05 \times 10^3 \times (400 - 18)$
$= 1,925$ MJ/min

6.

Energy supply	**Destination**	**Energy absorbed**	**Percentage**
12,6	Brake power	3,36	26,667%
	Coolant	5,426	43,067%
	Exhaust	1,925	15,28%
	Radiation	1,889	14,987%
Total		**12,6**	**100%**

Exercise 1.7

1. A four-cylinder, four-stroke petrol engine is tested on a Prony brake. The brake arm is 1 067 mm long and a load of 6,35 kg balances the brake at 3 000 rpm with a mechanical efficiency of 70%. The cooling water used is 6,2 kg/min., with a temperature rise of 43,3 °C. The air–fuel ratio is 19 : 1. The exhaust temperature rise is 538 °C, with a specific heat capacity of 1 kJ/kg.K. The fuel consumption is 0,142 kg per minute, with a heat value of 44 200 kJ/kg. The specific heat capacity of water is 4,187 kJ/kg.K.

Calculate the following:
1.1 brake power
1.2 indicated power
1.3 heat to cooling water per minute
1.4 heat to exhaust gas per minute
1.5 brake mean specific fuel consumption
1.6 Draw up a complete heat balance sheet in kJ/min and as a percentage

2. The brake mean specific fuel consumption of an IC engine is 0,3 kg/kWh at 3 000 r/min. The heat value of the fuel is 44 200 kJ/kg. Air–fuel ratio 19 : 1. Exhaust temperature rise is 732 °C. Friction power is 8,2 kW and the specific heat capacity of exhaust gas is 1,05 kJ/kg.K. Brake torque = 119 Nm and heat to cooling water is equal to heat developed by brake power.

Calculate the following:
2.1 brake power
2.2 fuel consumption in kg/min
2.3 heat from fuel per minute
2.4 heat to brake power per minute
2.5 mass of exhaust gas per minute
2.6 Draw up a heat balance sheet in kJ/min and as a percentage

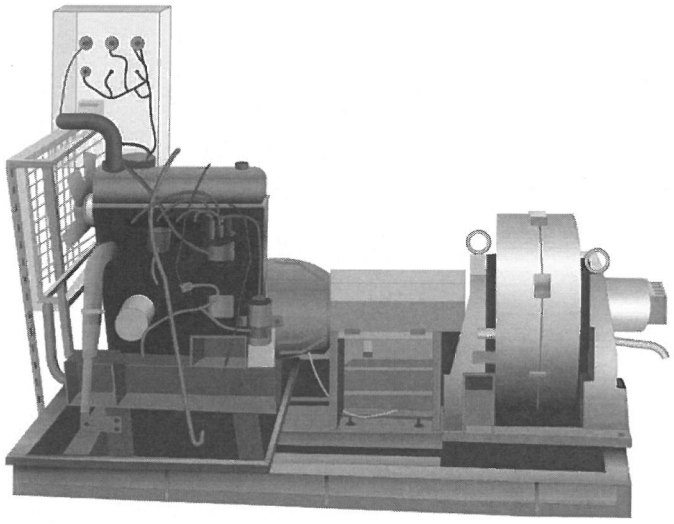


Figure 1.18: An IC engine test rig

Module summary
Volumes

- **Cylinder volume (V_1)**
  This is the total volume of the cylinder when it is at bottom dead centre.
- **Clearance volume (V_2)**
  This is the minimum volume in the cylinder when it is at top dead centre.
- **Stroke volume (V_s)**
  This is the difference between cylinder volume and clearance volume. It depends on the diameter and stroke length of the piston.

$$V_s = LA = V_1 - V_2$$

Compression ratio (r_c): It is the ratio between the cylinder volume and the clearance volume.

$$r_c = \frac{V_{cyl}}{V_c} = \frac{V_1}{V_2}$$

Powers

Fuel power: The power supplied by the fuel (input power)

$$P_f = \dot{m}_f \times CV$$

Indicated power: The power developed in the cylinder

$$P_i = p_{im} \times LANEx$$

Brake power: The power available on the crankshaft (output power)

$$P_b = p_{bm} \times LANEx$$

$$P_b = 2\pi NT$$

Friction power: The power lost due to friction in cylinder

$$P_\mu = P_i - P_b$$

Efficiencies

Air standard efficiency

This efficiency serves as a benchmark for evaluating the performance of real engines by comparing their actual efficiencies to the theoretical maximum efficiency achievable under ideal conditions

$$\eta_{as} = \frac{W}{Q_{in}}$$

or

$$\eta_{as} = 1 - \frac{Q_{out}}{Q_{in}}$$

Indicated thermal efficiency: It compares the indicated power in the cylinder to the fuel power (cylinder to fuel)

$$\eta_{it} = \frac{P_i}{P_f}$$

Brake thermal efficiency: It compares the brake power on the crankshaft to the fuel power (crank to fuel)

$$\eta_{bt} = \frac{P_b}{P_f}$$

Mechanical efficiency: It compares the brake power to the indicated power (crank to cylinder)

$$\eta_m = \frac{P_b}{P_i}$$

Indicated efficiency ratio: It compares the actual indicated efficiency to the ideal air standard efficiency

$$\eta_{ir} = \frac{\eta_{it}}{\eta_{as}}$$

Brake efficiency ratio: It compares the actual brake efficiency to the ideal air standard efficiency

$$\eta_{br} = \frac{\eta_{bt}}{\eta_{as}}$$

Indicated mean effective pressure: It is the average pressure during the complete cycle process

$$p_{im} = \frac{W}{V_s}$$

$$BSFC = \frac{m_f/h}{P_b}$$

$$ISFC = \frac{m_f/h}{P_i}$$

Gas cycles

- Otto

- Diesel
- Dual
- Joule

Engine tests

- Rope brake: Test to determine brake power
- Prony brake: Test to determine brake power
- Morse test: Test to determine indicated power
- Indicator diagram: test to determine indicated mean effective pressure

Energy balance

Energy heat balance is done on an internal combustion engine to determine the percentage heat converted into useful work (brake power) and the heat lost to the coolant, exhaust gasses and radiation.

Summative assessment

1. The volume–compression ratio for an Otto cycle = 8,5 : 1. The stroke length and diameter is 150 mm. Compression index= 1,33 and expansion index = 1,36. At the start of compression the pressure = 67 kPa and the temperature = 90 °C. The temperature at beginning of expansion = 1 440 K. Assume C_v = 0,72 kJ/kg.K. and the gas constant R = 0,286 kJ/kg.K.

Calculate the following:

1.1 temperatures at end of compression and expansion	[2]
1.2 pressures at the main points	[3]
1.3 heat supplied per kg of gas	[1]
1.4 work done per kg of gas	[3]
1.5 mean effective pressure	[4]

2. A single-cylinder, four-stroke petrol engine working on the Otto cycle has a volume-compression ratio of 6,5:1. The temperatures before and after the compression stroke is 300 K and 550 K respectively. The compression and expansion index is the same. The area enclosed by the PV diagram is 2,6 cm^2. The scale is as follows: 1 cm represents 675 kPa and 1 cm represents 0,012 m^3 on the PV diagram. The mass of air used is 0,04 kg per cycle and R = 288 J/kgK.

Calculate the following:

2.1 compression and expansion index	[4]
2.2 work done	[2]
2.3 temperatures before and after expansion	[6]

3. A single cylinder working on the diesel cycle takes in air at 90 kPa and 17 °C; maximum pressure in cycle = 3 378 kPa; fuel cut-off at 10% of stroke after TDC; compression and expansion curves follow the law $PV^{1,3}$= C; R = 288 J/kg.K; mass rate of air used = 0,46 kg/s. The crank rotates at 1 200 rpm.

Calculate the following:

3.1 temperature after compression	[1]
3.2 volumes at the main points in liters	[5]
3.3 pressure before exhaust stroke	[1]
3.4 power developed	[5]

4. A single cylinder engine working on the four-stroke diesel cycle develops a brake power of 11,4 kW at 1 500 r/min., mechanical efficiency = 85%, indicated mean effective pressure = 690 kPa. Pressure at beginning of the compression stroke = 96,5 kPa, stroke length is 1,5 times the piston diameter.; compression ratio = 15:1; expansion index = 1,25; compression index = 1,4; fuel is cut off at 10% of stroke after TDC; exhaust temperature = 1 580 K; C_p = 1 kJ/kg.K.

Calculate the following:

4.1 indicated power	[1]
4.2 piston diameter	[2]
4.3 volumes at all the change points in cm^3	[3]
4.4 pressures at the end of compression and expansion	[2]
4.5 temperatures at the change points	[3]
4.6 heat supplied by fuel per kg	[1]

5. The volume–compression ratio of a dual cycle is 22:1; stroke = 100 mm; cylinder diameter = 102 mm; compression and expansion index = 1,35;
volume–expansion ratio = 6,2:1. pressure at start of compression = 103 kPa and temperature = 370 K; exhaust pressure = 690 kPa.

Calculate the following:

5.1 volume at all main points in cm^3	[4]
5.2 pressures before and after constant volume heat supply	[2]
5.3 temperatures at all main points	[4]

6. A single-cylinder engine working on the dual cycle has a clearance volume of 1/17 of the stroke volume. Cylinder diameter = 78,5 mm; stroke = 99,5 mm; Air pressure at start of compression = 69 kPa =; initial temperature = 310 K.
Compression and expansion follow the law $PV^{1,38}$ = constant; R = 287 J/kg.K;
C_v = 718 J/kg.K. heat added during constant volume is 144 J. Mass rate of air used is 0,0004 kg per cycle; exhaust pressure = 345 kPa.

Calculate the following:

6.1 clearance and stroke volumes in cm^3	[3]
6.2 pressure and temperatures after compression	[2]
6.3 temperature and pressure after constant volume heat supply	[2]
6.4 temperatures after and before expansion	[2]
6.5 volume after constant pressure heat supply in cm^3	[1]
6.6) heat supplied per cycle	[3]
6.7 work done per cycle	[4]
6.8 thermal efficiency	[1]

7. In a gas turbine the temperature of the working fluid at inlet to the compressor is 17 °C and at outlet it is 167 °C. At inlet to the turbine the temperature is 727 °C and exhaust from the turbine at 467 °C. Compressor has a pressure ratio of 3,6:1; gamma = 1,4; C_p = 1,005 kJ/kg.K; R = 0,287 kJ/kgK. The mass flow rate of the gas is 90 kg/min.

Calculate the following:

7.1 net power	[3]
7.2 heat supplied per second	[1]
7.3 thermal efficiency of the plant	[1]
7.4 isentropic efficiency of the compressor	[2]
7.5 isentropic efficiency of the turbine	[2]

8. A gas turbine engine working on the Joule cycle has compression and expansion adiabatic efficiencies of 85%. P_1 = 103,4 kPa; P_2 = 620; 4 kPa; T_1 = 288 K; T_3 = 1 140 K; C_p = 1 kJ/kg.K; mass flow rate = 4,5 kg/s; γ = 1,4.

Calculate the following:

8.1 all adiabatic and actual temperatures	[4]
8.2 values of the polytropic indices of compression and expansion	[9]
8.3 heat supplied and heat rejected	[2]

9. A high performance four-cylinder petrol engine is tested on a dynamometer with the following results:

- Torque arm length 460 mm
- Maximum load on torque arm (at 5 000 r/min.) 620 N
- Load on torque arm at max power (6 800 r/min.) 550 N
- Fuel consumption (at 5 000 r/min.) 17,2 ml/s
- Density of the fuel is 735 kg/m^3
- Heat value of the fuel is 44 MJ/kg

Calculate the following:

9.1 maximum brake power [2]

9.2 specific fuel consumption in kg/kW/h [4]

9.3 brake thermal efficiency [2]

10. A five-cylinder, four-stroke petrol engine puts out 82,5 kW of brake power at an engine speed of 5 000 r/min. The indicated power is 100 kW. The volumetric compression ratio is 8,8. Each cylinder has a bore of 87,5 mm and a stroke of 80 mm. The air taken in amounts to 21 kg per kg of fuel consumed. The heat value of the fuel is 42 MJ/kg. The engine used fuel at a rate of 17,5 kg/h.
($R = 0{,}287$ kJ/kgK and $\gamma = 1{,}41$).

Determine the following:

10.1 indicated mean effective pressure [2]

10.2 mechanical efficiency [1]

10.3 volumetric efficiency at NTP [3]

10.4 brake thermal efficiency [2]

10.5 air standard efficiency [1]

10.6 brake efficiency ratio [1]

11. A five-cylinder petrol engine operates on the four-stroke cycle and during a test the following readings were taken:
cylinder diameter = 75 mm; stroke length = 75 mm;
mass on brake arm of 400 mm length = 20 kg at an engine speed of 3 600 r/min; indicated mean effective pressure = 700 kPa;
fuel consumption = 10,9 kg/h; heat value of the fuel = 44,5 MJ/kg.

Calculate the following:

11.1 brake power [3]

11.2 indicated power [1]

11.3 mechanical efficiency [1]

11.4 indicated thermal efficiency [2]

12. A two-stroke marine diesel engine has eight cylinders. The cylinders are 888 mm in diameter and the stroke length is 1,55 m. The clearance volume for each cylinder is 0,0735 m^3. The fuel has a heat value of 45,2 kJ/kg and the fuel consumption is 1 815 kg/h. The indicator cards has an average area of 805 mm^2 and a base length of 108 mm. The spring number is 81,5 kPa per mm. The engine speed is 120 rpm.

Calculate the following:

12.1 indicated power [3]

12.2 indicated thermal efficiency [2]

12.3 air standard efficiency [3]
12.4 indicated thermal efficiency ratio [1]

13. A double-acting two-stroke 8-cylinder diesel engine has a cylinder diameter of 450 mm and a stroke length of 680 mm. It was tested on a dynamometer at a speed of 200 r/min. Brake torque = 270 kNm; mechanical efficiency = 85%; heat value of fuel = 46,5 MJ/kg; air–fuel ratio = 30:1; brake thermal efficiency = 33%.

Calculate the following:

13.1 brake power [1]
13.2 indicated power [1]
13.3 indicated thermal efficiency [1]
13.4 mass of fuel used in kg/min [2]
13.5 indicated specific fuel consumption [1]
13.6 mass of exhaust gases in kg/min [1]
13.7 indicated mean effective pressure [2]
13.8 power lost due to friction [1]

14. The data below was obtained during a test on a single-cylinder four-stroke diesel engine.

- Duration of test: 60 minutes
- Stroke length of engine: 380 mm
- Cylinder diameter: 255 mm
- Total number of revs: 19 700
- Fuel used: 6,28 kg
- Heat value of oil: 44,5 MJ/kg
- Air to fuel ratio: 30:1
- Average area of indicator diagram: 568 mm^2
- Length of indicator diagram: 76 mm
- Spring number: 82 kPa per mm
- Brake torque: 760 Nm
- Cooling water used: 340 kg
- Temperature rise of cooling water: 45 °C
- Specific heat capacity for water: 4,187 kJ/kgK
- Temperature of exhaust gases: 400 °C
- Temperature of surroundings: 16 °C
- Mean specific heat capacity of exhaust gases: 1,05 kJ/kg.K

Calculate the following:

14.1 indicated power [3]

14.2 brake power [1]

14.3 mechanical efficiency [1]

14.4 brake thermal efficiency [2]

14.5 Draw up an energy balance on a basis of 1 minute and express each item as a percentage of heat supplied [7]

15. A single-acting four-stroke six-cylinder marine diesel engine has a stroke length of 910 mm and a cylinder diameter of 610 mm.

The results below were obtained during a test:

- Average area of indicator diagram = 7,3 cm^2
- Length of diagram = 7,7 cm
- Spring number = 67 kPa per mm height
- Brake torque at 130 r/min. = 62 kNm
- Fuel used = 240 kg/h with a heat value of 46 MJ/kg
- Cooling water flow through the engine = 200 kg/min., with a temperature rise of 55 °C and a specific heat capacity of 4,2 kJ/kg.K
- Air–fuel ratio = 30:1
- Specific heat capacity of exhaust gases = 1,005 kJ/kg.K
- Engine room temperature = 15 °C
- Temperature of exhaust gases = 330 °C

Calculate the following:

15.1 mechanical efficiency [5]

15.2 brake thermal efficiency [2]

15.3 Draw up an energy balance on a basis of 1 minute and express each item as a percentage of heat supplied [7]

16. In a trial on a four-cylinder petrol engine running at 2 150 r/min., the brake torque was 110 Nm. Each cylinder was cut out in turn and the speed returned to 2 150 r/min. The torque readings are tabulated below:

Cylinder number	1	2	3	4
Torque (in Nm)	72,35	74,18	75,67	73,23

These results were also obtained:

- fuel used = 0,26 kg per brake kilowatts per hour
- heat value of fuel= 45 MJ/kg
- specific heat capacity of exhaust gases = 1,038 kJ/kg.K
- temperature rise of exhaust gases = 510 °C

- cooling water flow = 4,58 kg/min., with a temperature rise of 39 °C
- specific heat capacity of water = 4,18 kJ/kg.K.
- Air-fuel ratio is 25:1

Calculate the following:

16.1 mechanical efficiency [8]

16.2 mass of fuel used per minute [1]

16.3 heat supplied by fuel per minute [1]

16.4 heat lost to exhaust gases per minute [2]

16.5 heat lost to coolant per minute [1]

16.6 Draw up an energy balance on a basis of 1 minute and express each item as a percentage of heat supplied. [4]

Total: 200 marks

MODULE

2 *Air compressors*

By the end of this module, students should be able to:

- draw a clearly labelled pressure versus volume diagram showing all principal points for a multistage compressor with or without clearance volume;
- perform calculations at all change points for volumes, temperatures, pressures, work done, power, volumetric and isothermal efficiencies according to compression and constant pressure processes:
  - with maximum efficiency
  - without complete intercooling; and
- determine isothermal work done using relevant formula.

Introduction

Air compressors are useful and necessary tools in industry. Mechanical work is converted to heat in the compressor, so they are heat engines

The function of air compressors

An air compressor takes in a definite quantity of fluid (usually a gas, most often air) and delivers it at a required pressure. An efficient compressor will accomplish this with the minimum input of mechanical work.

Air compression is a direct application of the gas laws. By making a few simplified assumptions about the behaviour of the air in the cylinder, it is possible to estimate the performance of the machines reasonably accurate.

Types of compressors

Air compressors fall into two distinct types. There are important differences in the fundamental behaviour of the two types, the reciprocating compressor and the rotary (or centrifugal) compressor.

On a performance basis, a general distinction can be made between the two types.

- The reciprocating compressor has the characteristics of a low mass rate of flow and high pressure ratios.
- The rotary compressor has a high mass rate of flow and low pressure ratios.

Some rotary machines are suitable for low-pressure work only. They are applied to the scavenging and supercharging of engines, and the various applications of exhausting and vacuum pumping. For

pressures above 900 kPa, the vane-type rotary compressor could be used to supply boost pressures, but for sustained high-pressure work of 48,5 MPa and above for special purposes, the reciprocating compressor is used.

Both basic types are available in different forms, each with its own characteristics. They could be single- or multi-stage, and have either air or water cooling.

The single-stage, single-acting air compressor

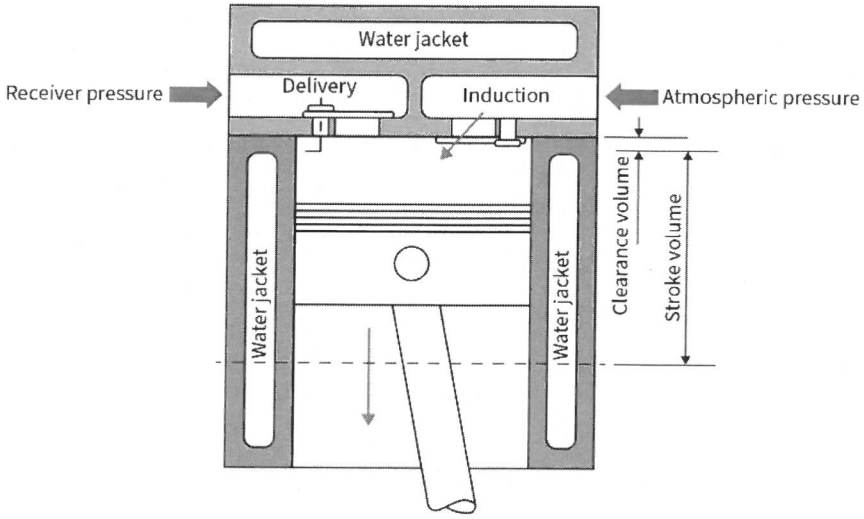


Figure 2.1: A single-stage, single-acting air compressor

The single-stage, double-acting air compressor

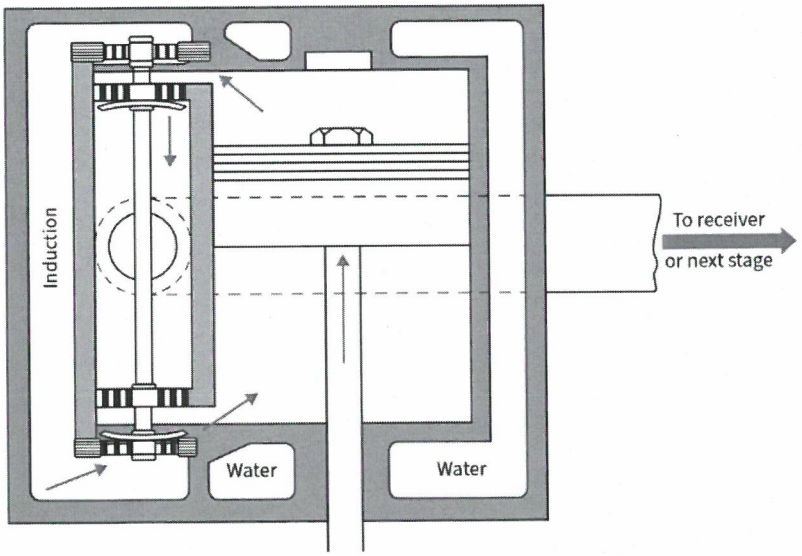


Figure 2.2: A single-stage, double-acting air compressor

2.1 Reciprocating air compressors

The reciprocating compressor in the diagram below is in its simplest form.

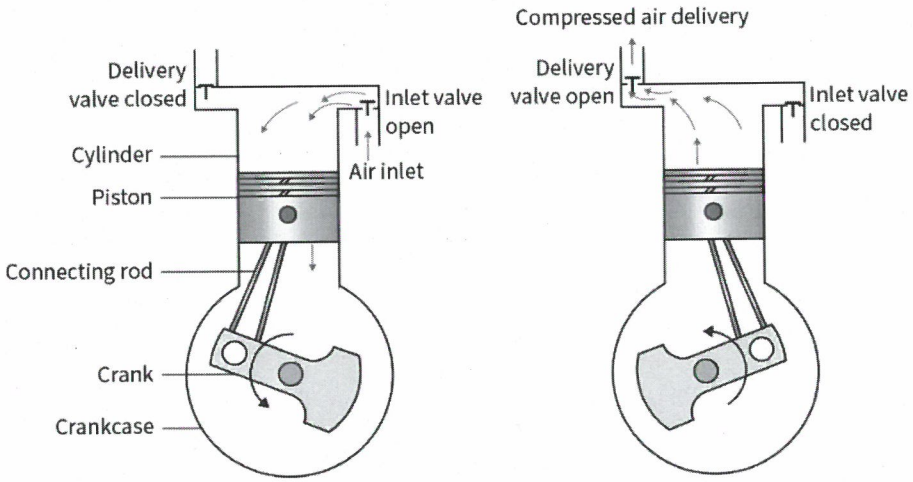


Figure 2.3: Reciprocating compressor

It consists of a piston that reciprocates in a cylinder, driven through a connecting rod and crank-mounted in a crankcase. There are inlet and delivery valves mounted in the head of the cylinder. These valves are usually of the pressure differential type, meaning that they will operate as a result of difference in pressure across the valve.

No elaborate valve-operating mechanism is necessary. The valves used in most compressors are designed to give automatic action. They are a spring-loaded type, operated by a small pressure difference across them, the light spring pressure giving a rapid-closing action. The lift of the valve to give the required airflow should be as small as possible, and should operate without shock.

The air or gas is usually drawn into the cylinder during the outward stroke of the piston, and compressed and delivered during the inward stroke of the piston.

Since air is locked up in the cylinder of a reciprocating compressor, the compression pressure for this type of compressor could be very high. The pressure is limited by the strength of the various parts of the compressor and the power of the driving motor.

Note
With the reciprocating compressor there is an intermitted flow of air.

The next diagram shows a theoretical P-V diagram for a reciprocating air compressor neglecting clearance.

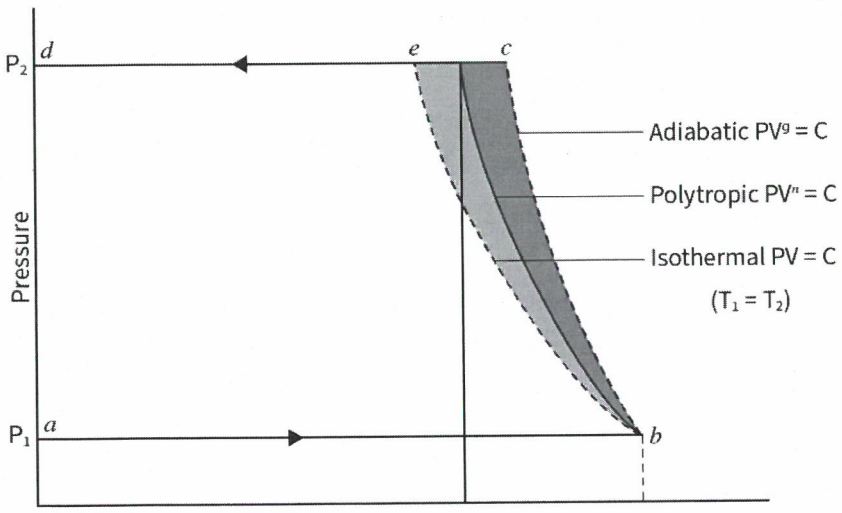


Figure 2.4: PV diagram for reciprocating air compressor

The constant pressure line *ab* represents air being drawn into the cylinder at volume V_1, pressure P_1 and temperature T_1. The air is then compressed adiabatically from *b* to *c* from pressure P_1 to pressure P_2. Volume decreases from V_1 to V_2, and the temperature increases from T_1 to T_2.

Air is forced out of the cylinder at constant pressure from *c* to *d* with volume V_2, and at pressure P_2 with temperature T_2 delivered from the compressor. The work done is represented by the area *abcd*.

An isothermal line has more slope than an adiabatic line, so if the air had been compressed isothermally, it would have followed the line *be*. The work done on the air would be the area *abed*, which is considerably less due to the adiabatic compression.

The most efficient way to compress air is isothermally. This is not possible in practice, as an isothermal compression would mean that the compressor would need to run extremely slow.

In practice, the compressors are driven at fairly high speeds in order to compress as much air as possible in a given time, so the compression of the air will approximate an adiabatic.

In the diagram, the work saved by compressing isothermally is shaded, and it is clear how important it is to attempt to approximate this isothermal compression in order to save this large amount of work. Two methods are used for approximating to the isothermal compression while running the compressor at high speed.

Method 1: Cold water spray

In this method, cold water is sprayed into the cylinder during compression, reducing the temperature of the air. Without the cold water spray, the compression would have been adiabatic (PV^{γ} = constant). The spray effectively reduces the compression to the polytrophic form PV^{n} = constant. This effect is clearly shown in the diagram above. It has brought the compression line closer to the isothermal, thus saving a considerable amount of work.

Method 2: Multi-stage compression

The air is compressed in several stages, instead of full compression being performed in a single cylinder. In principle it is equivalent to a number of compressors in series, the air passing from one cylinder to the next, and the pressure increasing in each cylinder.
The diagram shows the P-V diagram for a single stage compressor.

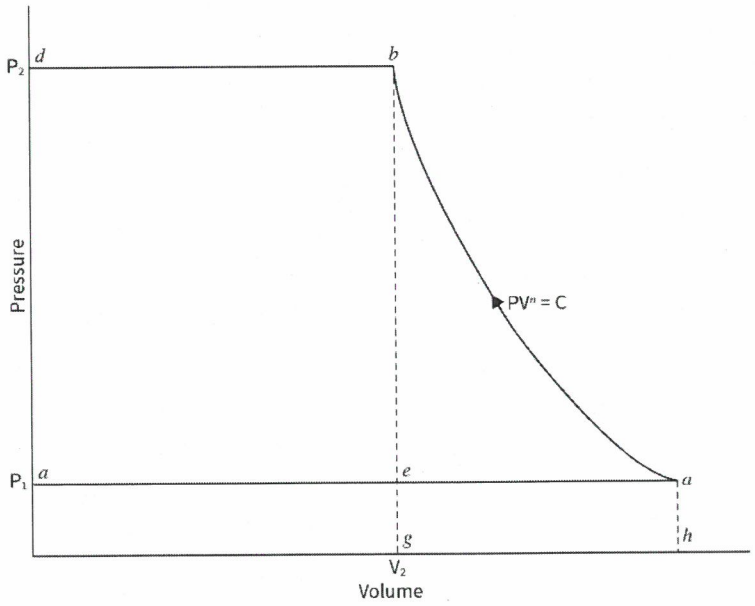


Figure 2.5: PV diagram for single-stage compressor

Let P_1 and V_1 be the initial condition, and P_2 and V_2 the final conditions of the compressor.

Work required per cycle = area $abcd$

$$= (\text{area } ebgf) + (\text{area } gbah) - (\text{area } adfh)$$

$$= (P_2V_2) + \frac{(P_2V_2)-(P_1V_1)}{n-1} - (P_1V_1)$$

LCM = n − 1

Worked required per cycle = $\frac{(nP_2V2-P_2V_2)+(P_2V_2-P_1V_1)-(nP_1V_1-P_1V_1)}{n-1}$

$= \frac{nP_2V_2-P_1V_1}{n-1}$

And taking P_1V_1 outside the brackets:

W/cycle= $P_1V_1\frac{n}{n-1}(\frac{P_2V_2}{P_1V_1}-1)$

.....①

However, for a polytrophic compression:

$P_1V_1^n = P_2V_2^n$

$\therefore \frac{V_2}{V_1} = \left(\frac{P_2}{P_1}\right)^{-\frac{1}{n}}$

.....②

Substituting equation ② into ①:

$P_1V_1^n\frac{n}{n-1}\left[\frac{P_2}{P_1}\times\left(\frac{P_2}{P_1}\right)^{-\frac{1}{n}}-1\right]$

$P_1V_1^n\frac{n}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{-\frac{n-1}{n}}-1\right]$ joules/cycle

OR

$mRT_1\frac{n}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{-\frac{n-1}{n}}-1\right]$ joules/cycle

The effect of clearance on a reciprocating compressor

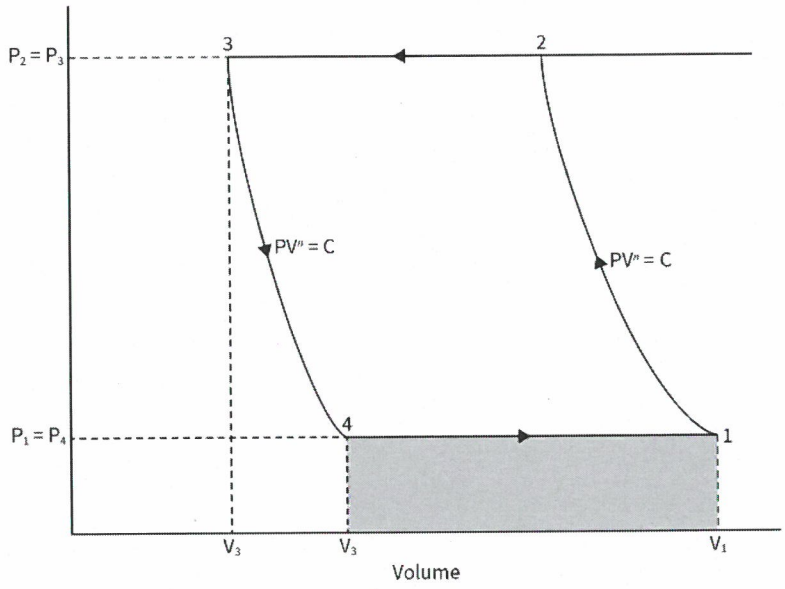


Figure 2.6

The clearance volume on the cylinder of the compressor should be as small as possible, since it seriously affects the volumetric efficiency.

Let V_3 represent the clearance volume in the P-V diagram above, which shows that the air left in the clearance space at the end of the complete compression and exhaust stroke will expand to V_4 at the beginning of the suction stroke. The law of expansion will also be PV^n = constant. This means that the new air admitted during the remainder of the suction stroke can only occupy volume ($V_1 - V_4$). Thus there is a considerable reduction in volumetric efficiency due to the clearance volume. Because of this, it is usual in practice to limit the clearance volume to not more than 4% to 6% of the swept volume.

The work done on the air delivered is not affected by the clearance, as the work required to compress the clearance volume of air is theoretically regained during its expansion from V_3 to V_4.
Below is a theoretical P–V diagram for a reciprocating air compressor, neglecting clearance.

Note
If the clearance volume is not mentioned in the problem, V1 would be the
swept volume or stroke volume only.
However, if clearance volume is mentioned, V1 would be the swept volume plus the clearance volume.

$$W/cycle = P_1(V_1 - V_4)\frac{n}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right] kJ/cycle$$

Revision of single stage compressors
Before you attempt any calculations on compressors, you need to make sure that you understand the following concepts:

How to convert volume, mass and work done to volume rate, mass rate
and power

$$\dot{V} = V \times NE$$

$$\dot{m} = m \times NE$$

$$P = W \times NE$$

Where:
N is rotational speed of crank (rps)
E is 1 for single acting and 2 for double acting cylinder

The difference between stroke volume, effective volume, clearance volume and cylinder volume

$V_s = LA$

$V_e = V_s \times \eta_v$

Where

η_v is volumetric efficiency

$$\eta_v = 1 - \frac{V_c}{V_s}\left[\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} - 1\right]$$

$V_{cyl} = V_s + V_c$

The difference between total mass, effective mass and mass remaining in cylinder

Total mass in cylinder (m_t)

$m_t = \frac{P_1V_1}{RT_1} = \frac{P_2V_2}{RT_2}$ (working with total volumes)

Mass drawn in or delivered (m_e)

$m_e = \frac{P_1V_{1e}}{RT_1} = \frac{P_2V_{2e}}{RT_2} = \frac{P_aV_a}{RT_a}$ (working with effective volumes)

Mass remaining after delivery (m_r)

$m_r = m_t - m_e$

Where

V_e is effective volume drawn in or delivered

P is pressure at start of compression or delivery

T is temperature at start of compression or delivery

Isothermal work done or power

$W_i = PV\ln\left(\frac{P_2}{P_1}\right)$

Compressor work done or power

$$W_c = \frac{nP_1V_1}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

Isothermal efficiency

$$\eta_i = \frac{W_i}{W_c} = \frac{P_i}{P_c}$$

Volumetric efficiency at NTP

$$\eta_{va} = \frac{V_a}{V_s}$$

Where

V_a is free air delivery

Volumetric efficiency at inlet

$$\eta_v = \frac{V_e}{V_s}$$

Example 2.1

A single acting reciprocating compressor has a cylinder diameter of 140 mm and stroke length of 224 mm. The clearance volume is 5% of the stroke volume resulting in a volumetric efficiency of 88%. The pressure ratio is 5. The compression and expansion index is 1,3.

Calculate the following:

1. stroke volume
2. clearance volume
3. cylinder volume
4. volume drawn in
5. volume after compression
6. volume delivered

Solutions

1.

$$V_s = LA = 0{,}224 \times \pi \times 0{,}07^2 = 3{,}448 \times 10^{-3}\ m^3$$

2.

$$V_c = 0{,}05 \times V_s = 0{,}05 \times 3{,}448 \times 10^{-3} = 172{,}411 \times 10^{-6}\ m^3$$

3.

$$V_1 = V_s + V_c = 3{,}621 \times 10^{-3}\ m^3$$

4.

$$V_{e1} = V_s \times \eta_v = 3{,}448 \times 10^{-3} \times 0{,}88 = 3{,}034 \times 10^{-3}\ m^3$$

5.

$$V_2 = V_1 \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}} = 3{,}621 \times 10^{-3} \times \left(\frac{1}{5}\right)^{\frac{1}{1{,}3}} = 1{,}05 \times 10^{-3}\ m^3$$

6.

$$V_{e2} = V_{e1} \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}} = 3{,}034 \times 10^{-3} \times \left(\frac{1}{5}\right)^{\frac{1}{1{,}3}} = 0{,}88 \times 10^{-3}\ m^3$$

Example 2.2

A single acting reciprocating air compressor has a stroke volume of 5 litres at an absolute temperature of 300 K and a pressure of 100 kPa. The clearance volume is 6% of the stroke volume resulting in a volumetric efficiency of 85%. The gas constant for air is 288 J/kgK.

Calculate the following:

1. total mass in the cylinder after the inlet stroke
2. effective mass that was drawn in
3. mass remaining in the cylinder after delivery

Solutions

1.

$$V_c = 0{,}06 \times V_s = 0{,}06 \times 5 \times 10^{-3} = 300 \times 10^{-6}\ m^3$$

$$V_1 = V_c + V_s = 300 \times 10^{-6} + 5 \times 10^{-3} = 5{,}3 \times 10^{-3}\ m^3$$

$$m_t = \frac{P_1 V_1}{RT_1} = \frac{100 \times 10^3 \times 5{,}3 \times 10^{-3}}{288 \times 300} = 6{,}134 \times 10^{-3}\ kg$$

2.

$$V_{e1} = V_s \times \eta_v = 5 \times 10^{-3} \times 0{,}85 = 4{,}25 \times 10^{-3}\ m^3$$

$$m_e = \frac{P_1 V_{e1}}{RT_1} = \frac{100 \times 10^3 \times 4{,}25 \times 10^{-3}}{288 \times 300} = 4{,}919 \times 10^{-3}\ kg$$

3.

$$m_r = m_t - m_e = (6{,}134 - 4{,}919)10^{-3} = 1{,}215 \times 10^{-3}\ kg$$

Important

In order to calculate the work done or power for a compressor with a given clearance volume, it is important to remember that the volume used in all the calculations must be the effective volume and not the stroke or cylinder volumes.

Example 2.3

A single acting reciprocating compressor has a stroke length of 250 mm and diameter of 100 mm. The air is drawn in at a pressure of 95 kPa and temperature of 17°C and is compressed to a pressure of 400 kPa. The compression and expansion index is 1,3. The clearance volume is 4% of the stroke volume. Assume R for air as 288 J/kgK.

Calculate the following:

1. stroke volume
2. volumetric efficiency
3. effective volume at inlet
4. absolute delivery temperature
5. work done by the compressor
6. isothermal efficiency

Solutions

1.

$$V_s = LA = 0{,}25 \times \pi \times 0{,}05^2 = 1{,}963 \times 10^{-3}\ m^3$$

2.

$$\eta_v = 1 - \frac{V_c}{V_s}\left[\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} - 1\right]$$

$$= 1 - 0{,}04\left[\left(\frac{400}{95}\right)^{\frac{1}{1{,}3}} - 1\right]$$

$$\eta_v = 0{,}919 = 91{,}913\%$$

3.

$$V_{e1} = V_s \times \eta_v = 1{,}963 \times 10^{-3} \times 0{,}919 = 1{,}805 \times 10^{-3}\ m^3$$

4.

$$T_2 = T_1 \times \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = 290 \times \left(\frac{400}{95}\right)^{\frac{1{,}3-1}{1{,}3}} = 404{,}088\ K$$

5.

$$W_c = \frac{nP_1V_{e1}}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{1{,}3 \times 95 \times 10^3 \times 1{,}805 \times 10^{-3}}{1{,}3 - 1}\left[\left(\frac{400}{95}\right)^{\frac{0{,}3}{1{,}3}} - 1\right]$$

$W_c = 292{,}276\ J$

6.

$$W_i = P_1 V_{e1} \ln\left(\frac{P_2}{P_1}\right)$$

$$= 95 \times 10^3 \times 1{,}805 \times 10^{-3} \times \ln\left(\frac{400}{95}\right)$$

$W_i = 246{,}47\ J$

$$\eta_i = \frac{W_i}{W_c} = \frac{246{,}47}{292{,}276} = 0{,}843 = 84{,}328\%$$

Example 2.4

A single acting reciprocating compressor has a stroke length of 200 mm and diameter of 120 mm. The air is drawn in at a pressure of 98 kPa and temperature of 27°C and compressed to a pressure of 500 kPa. The compression and expansion index is 1,25. The compressor is driven by a motor with a power of 8 kW at 1 200 rpm. The clearance volume is 6% of the stroke volume. Assume R for air as 288 J/kgK.

Calculate the following:

1. volume rate at inlet in litres/sec
2. absolute temperature after compression
3. volume rate delivered in litres/sec
4. power developed by the compressor
5. isothermal power
6. isothermal efficiency
7. mechanical efficiency

Solutions

1.

$$\dot{V}_s = \text{LANE}$$
$$= 0{,}2 \times \pi \times 0{,}06^2 \times 20 \times 1$$
$$\dot{V}_s = 45{,}239 \times 10^{-3}\ \text{m}^3/\text{s} = 45{,}239\ \ell/\text{s}$$

$$\eta_v = 1 - \frac{V_c}{V_s}\left[\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} - 1\right]$$
$$= 1 - 0{,}06\left[\left(\frac{500}{98}\right)^{\frac{1}{1{,}25}} - 1\right]$$
$$\eta_v = 0{,}839 = 83{,}902\%$$

$$\dot{V}_{e1} = \dot{V}_s \times \eta_v$$
$$= 45{,}239 \times 10^{-3} \times 0{,}839$$
$$\dot{V}_{e1} = 37{,}957 \times 10^{-3}\ \text{m}^3/\text{s} = 37{,}957\ \ell/\text{s}$$

2.

$$T_2 = T_1 \times \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = 300 \times \left(\frac{500}{98}\right)^{\frac{1{,}25-1}{1{,}25}} = 415{,}595\ \text{K}$$

3.

$$\dot{m}_e = \frac{P_1 \dot{V}_{e1}}{RT_1}$$
$$= \frac{98 \times 10^3 \times 37{,}957 \times 10^{-3}}{288 \times 300}$$
$$\dot{m}_e = 43{,}053 \times 10^{-3}\ \text{kg/s}$$

$$\dot{V}_{e1} = \frac{\dot{m}_e R T_2}{P_2}$$
$$= \frac{43{,}053 \times 10^{-3} \times 288 \times 415{,}595}{500 \times 10^3}$$
$$\dot{V}_{e1} = 10{,}306 \times 10^{-3}\ \text{m}^3/\text{s} = 10{,}306\ \ell/\text{s}$$

4.

$$P_c = \frac{nP_1\dot{V}_{e1}}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]P_i$$

$$= \frac{1{,}25 \times 98 \times 10^3 \times 37{,}957 \times 10^{-3}}{1{,}25 - 1}\left[\left(\frac{500}{98}\right)^{\frac{1{,}25-1}{1{,}25}} - 1\right]$$

$$P_c = 7{,}166\ \text{kW}$$

5.

$$P_i = P_1\dot{V}_{e1}\ln\left(\frac{P_2}{P_1}\right)$$

$$= 98 \times 10^3 \times 37{,}957 \times 10^{-3} \times \ln\left(\frac{500}{98}\right)$$

$$P_i = 6{,}062\ \text{kW}$$

6.

$$\eta_i = \frac{P_i}{P_c} = \frac{6{,}062}{7{,}166} = 0{,}846 = 84{,}587\%$$

7.

$$\eta_m = \frac{P_c}{P_m} = \frac{7{,}166}{8} = 0{,}896 = 89{,}58\%$$

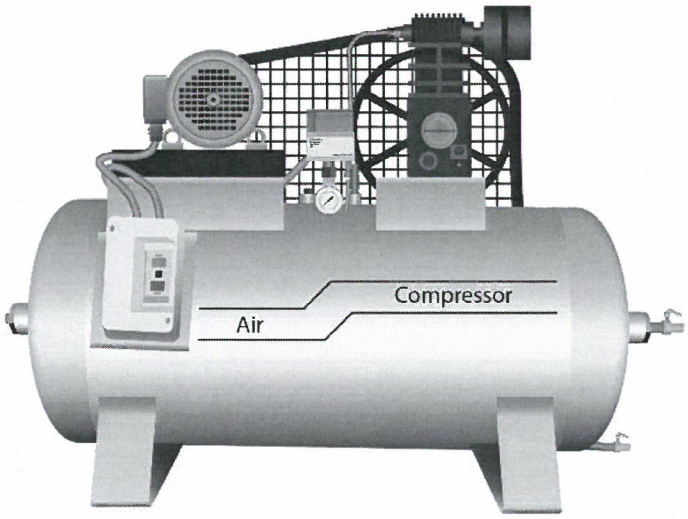


Figure 2.7 A reciprocating single-cylinder, single-acting air compressor

Exercise 2.1

1. A single acting reciprocating air compressor has a stroke length of 450 mm and cylinder diameter of 240 mm. The air is drawn in at a pressure of 95 kPa and temperature of 7°C and is then compressed to a pressure of 665 kPa. The compression and expansion index is 1,25. The clearance volume is 4% of the stroke volume. Assume R for air as 288 J/kgK.

Calculate the following:

1.1 stroke volume in litres

1.2 volumetric efficiency

1.3 effective volume at inlet in litres

1.4 free air delivery at NTP in litres

1.5 work done by the compressor

1.6 isothermal efficiency.

2. A double acting reciprocating compressor with a pressure ratio of 4 is required to draw in air at 100 kPa and 25°C at a rate of 0,6 m^3/sec. The compression and expansion index is 1,2. Assume that the clearance volume is 5% of the 60 litre stroke volume. Assume that the gas constant for air is 288 J/kgK. The drive is 85% efficient.

Calculate the following:
2.1 volumetric efficiency
2.2 speed of the crank
2.3 cylinder volume in litres
2.4 mass rate drawn in
2.5 mass remaining in cylinder after delivery
2.6 power to drive the compressor.

3. A single acting reciprocating compressor has a stroke length to diameter ratio of 2:1. It draws in a volume rate of 42 m^3/m at a pressure of 100 kPa and a temperature of 22°C. The delivery temperature is 157°C and the valve is set to open at 650 kPa. The compression index is 1,28. The crank rotates at 1 680 rpm. The mechanical efficiency is 80%. The volumetric efficiency is 85%.

Calculate the following:
3.1 diameter of the cylinder
3.2 cylinder volume in litres
3.3 compression index
3.4 power to drive the compressor.

4. A double acting reciprocating air compressor has a free air delivery of 500 litres/sec at NTP. The air is drawn in at a pressure of 97 kPa and temperature of 27°C and then compressed to a pressure of 485 kPa. The cylinder diameter is 200 mm. The compressor is

driven by a 125 kW motor at 1 200 rpm. Clearance volume is 6% of the stroke volume. The index for compression and expansion is 1,35.

Calculate the following:
4.1 volume rate at inlet in litres/sec
4.2 stroke length
4.3 volume rate delivered in litres/sec
4.4 mechanical efficiency
4.5 isothermal efficiency.

5. A double acting compressor draws in 15,6 m^3/min at a pressure of 95 kPa and 20°C. The compression ratio is 8:1. The cylinder volume: clearance volume ratio is 21:1. The index for compression and expansion is 1,3. The crank rotates at 600 rpm. Assume R for air as 288 J/kgK. The mechanical efficiency is 87%.

Calculate the following:
5.1 cylinder volume in litres
5.2 volume after compression in litres
5.3 volume after expansion in litres
5.4 motor power to drive the compressor
5.5 isothermal efficiency.

2.1.1 PV diagrams for multi-stage compressors

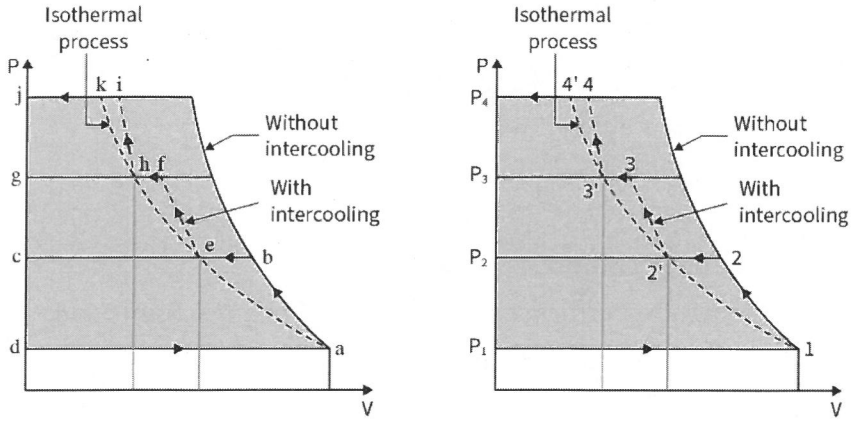


Figure 2.8: Multistage without clearance volume

Process **abcd**-first stage (low pressure stage)
Process **efgc**-second stage (intermediate pressure stage)
Process **hijg**-third stage (high pressure stage)
Process **be**-Intercooling between stages 1 and 2
Process **fh**-Intercooling between stages 2 and 3
Process **ik**-After cooling before delivery

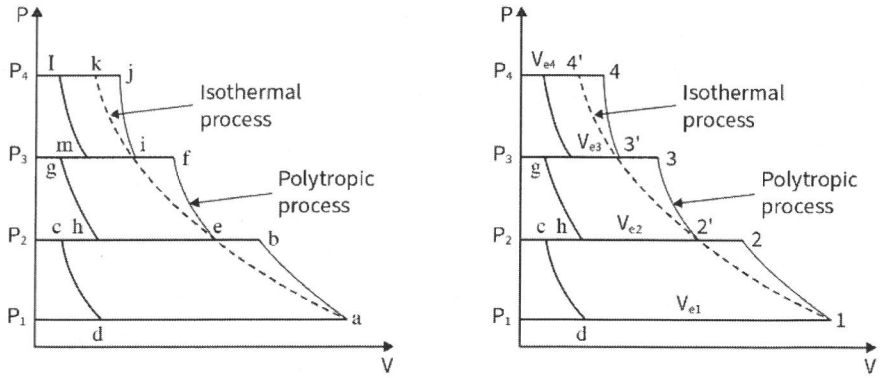


Figure 2.9: Multi stage with clearance volume

Process **abcd**-first stage (low pressure stage)
Process **efgh**-second stage (intermediate pressure stage)
Process **ijlm**-third stage (high pressure stage)
Process **be**-Intercooling between stages 1 and 2
Process **fi**-Intercooling between stages 2 and 3
Process **jk**-After cooling before delivery

2.1.2 Multi-stage compressors calculations

Here we'll learn to perform calculations at all change points and determine the isothermal work done.

The multi-stage reciprocating air compressor

The condition for minimum work is that the compression process should be isothermal. The delivery temperature increases with the pressure ratio. As the pressure ratio increases, the volumetric efficiency decreases.

The volumetric efficiency can be improved by carrying out the compression in two stages. After the first stage compression, the air is passed into a smaller cylinder, where the air is compressed to the final required pressure. If the machine has two stages, the air would be delivered at the end of this stage, but it could be delivered to a third cylinder for higher pressure ratios. The cylinders of the successive stages are proportional to taking the volume of gas or air delivered from the previous stage.

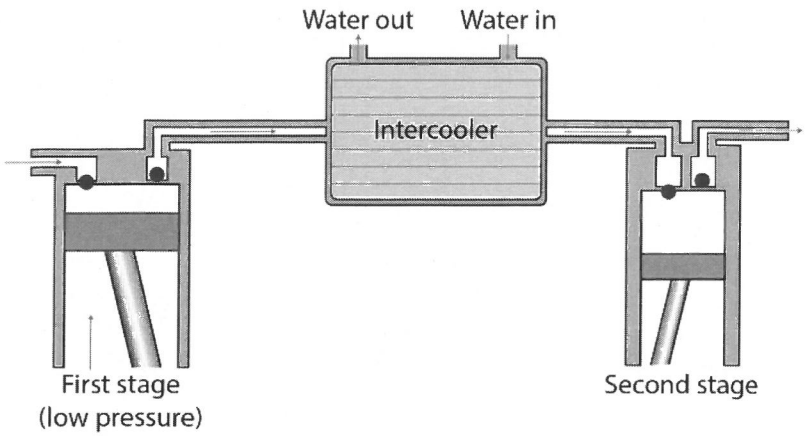


Figure 2.10

The diagram shows a first-stage compressor, an intercooler and a second-stage compressor, where the delivery process from the first- or low-pressure stage, and the inlet pressure of the second or high pressure stage are at the same pressure.

Ideal isothermal compression can only be reached if ideal cooling is continuous, which is difficult during normal compression. With multi-stage compression, gas can be cooled by passing it through an intercooler as it is transferred from one cylinder to the next. When intercooling is complete, the air will enter the second stage at the same temperature as when it entered the first stage. This is called **complete intercooling.**

Work done by a two-stage compressor

In the diagram on the next page, the total work done is the shaded area BCHEFAB.

The air is compressed from B to C, then cooled until its temperature drops to the original temperature at H (cooled at constant pressure

until it reaches the isothermal line at H). Therefore H is on the isothermal line BHG. From H, further compression to point E takes place.

Important terminology that is important to understand when working with multi-stage compressors.

Ratios with geometric progression

This means that all the stages have equal pressure ratios.

$$\frac{P_2}{P_1} = \frac{P_3}{P_2} = r_p = \sqrt[x]{\frac{P_{x+1}}{P_1}}$$

Where

x is number of stages

Compressor with perfect intercooling

This means that the temperature after compression in each stage is cooled down in an intercooler to the original temperature, before it is compressed in the next stage.

$$T_{2'} = T_1$$

Compressor is designed for maximum efficiency

This means the pressure ratios are the same for all stages and the temperatures at the start of compression for each stage is the same (perfect intercooling). The work done or power for each stage will then be the same. We can therefore calculate the work done or power for the first stage only and then multiply the answer with the number of stages (x) to determine the total work done or power.

Pressure stages are not equal or intercooling is incomplete

This means that each stage will have its own pressure ratios and the work done or power needs to be calculated individually for each stage and then added to determine the total.

Heat absorbed by the intercoolers and after cooler

There is an intercooler between each stage and sometimes an after cooler at the end of the last stage. The heat absorbed by each of these components can be calculated. This process takes place at constant pressure.

$$Q_{ic} = m_e C_p (T_2 - T_{2'})$$

Heat transfer to water jackets

This is the heat that is absorbed by the water in the water jackets that surround the cylinder.

$$Q_{wj} = W_c - Q_{ic}$$

or

$$Q_{wj} = m_e \left(\frac{C_p - nC_v}{n-1}\right)(T_2 - T_1)$$

Remember to always calculate the work done, heat transfer to intercooler and heat transfer to water jacket for one stage only. These values can then each be multiplied by their respective numbers to calculate the totals if required.

Effective mass and effective volumes for each stage

Remember that each stage will have its own clearance volume, effective volume, stroke volume and cylinder volume which means that each stage will also have its own volumetric efficiency.

Consider each stage separately when doing calculations.

$$m_e R = \frac{P_1 V_{e1}}{T_1} = \frac{P_2 V_{e2}}{T_{2'}} = \frac{P_3 V_{e3}}{T_{3'}} = \frac{P_a V_a}{T_a}$$

The volumes are the effective volumes drawn into each cylinder and the temperatures are the temperatures at the start of each compression. The top volume on the diagram will be the effective volume that is delivered and the temperature at the start of the delivery stroke will determine the delivered volume. The effective mass (m_e) and the gas constant stays constant throughout.

Example 2.6

A three-stage compressor delivers 6,4 MPa air pressure from an inlet pressure of 100 kPa. The temperature at start of compression is 27 °C. The index of compression and expansion for all stages is 1,2. The compressor is designed for minimum work. The gas constant R is 287 J/kgK and C_p is 1,005 kJ/kgK.

Calculate the following:

1. intermediate pressures
2. temperature at end of compression for each stage
3. heat extracted per kg in each intercooler
4. work done per kg
5. heat absorbed per kg in each water jacket
6. isothermal work done per kg
7. isothermal efficiency

Solutions

1.

$$r_p = \sqrt[x]{\frac{P_{x+1}}{P_1}} = \sqrt[3]{\frac{6{,}4 \times 10^6}{100 \times 10^3}} = 4$$

$$P_2 = 4 \times P_1 = 400 \text{ kPa}$$

$$P_3 = 4 \times P_2 = 1{,}6 \text{ MPa}$$

2.

$$T_2 = T_1 \times \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = 300 \times (4)^{\frac{0{,}2}{1{,}2}} = 377{,}976 \text{ K}$$

$$T_2 = T_3 = T_4 = 377{,}976 \text{ K}$$

3.

$$Q_{ic} = m_e C_p (T_2 - T_{2'})$$
$$= 1 \times 1{,}005(377{,}976 - 300)$$
$$Q_{ic} = 78{,}366 \text{ kJ/kg}$$

4.

$$W_c = \frac{nmRT_1}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$
$$= \frac{1{,}2 \times 1 \times 287 \times 300}{0{,}2}\left[(4)^{\frac{0{,}2}{1{,}2}} - 1\right]$$
$$W_c = 134{,}275 \text{ kJ/kg}$$

$$W_{ct} = x \times W_c = 3 \times 134{,}275 = 402{,}826 \text{ kJ/kg}$$

5.

$$Q_{wj} = W_c - Q_{ic} = 134{,}275 - 78{,}366 = 55{,}909 \text{ kJ/kg}$$

6.

$$W_i = mRT_1 \ln\left(\frac{P_2}{P_1}\right)$$
$$= 1 \times 287 \times 300 \times \ln(4)$$
$$W_i = 119{,}36 \text{ kJ/kg}$$

$$W_{it} = x \times W_i = 3 \times 119{,}36 = 358{,}08 \text{ kJ/kg}$$

6.7)

$$\eta_i = \frac{W_i}{W_c} = \frac{119{,}36}{134{,}275} = 0{,}889 = 88{,}892\%$$

Example 2.7

A three-stage, single acting reciprocating compressor takes in air at a rate of 14,4 m^3/min at a pressure of 100 kPa and 27°C. The delivery pressure is 2,7 MPa. The stage pressures are in geometric progression and intercooling is perfect. The compression and expansion coefficient is 1,32 for all stages.. Each stage has a volumetric of 96% and the clearance volumes are constant. The crank rotates at 200 rpm. The gas constant R is 287 J/kgK.

Calculate the following:

1. pressure ratio
2. stroke volume of the low pressure cylinder
3. mass rate delivered per minute
4. effective volume of the second stage
5. volume delivered per minute if no after cooler
6. isothermal efficiency

Solutions

1.

$$r_p = \sqrt[x]{\frac{P_{x+1}}{P_1}} = \sqrt[3]{\frac{2{,}7 \times 10^6}{100 \times 10^3}} = 3$$

2.

$$V_{e1} = \frac{\dot{V}_{e1}}{NE} = \frac{14{,}4}{200 \times 1} = 72 \times 10^{-3}\ m^3$$

$$V_s = \frac{V_{e1}}{\eta_{v1}} = \frac{72 \times 10^{-3}}{0{,}96} = 75 \times 10^{-3}\ m^3$$

$$\eta_v = 1 - \frac{V_c}{V_s}\left[\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} - 1\right]$$

$$0{,}96 = 1 - \frac{V_c}{0{,}075}\left[(3)^{\frac{1}{1{,}32}} - 1\right]$$

$$V_c = 2{,}31 \times 10^{-3}\ m^3$$

$$V_{cyl1} = V_s + V_c = 77{,}31 \times 10^{-3}\ m^3$$

3.

$$\dot{m}_e = \frac{P_1 \dot{V}_{e1}}{RT_1}$$

$$= \frac{100 \times 10^3 \times 14{,}4}{287 \times 300}$$

$$\dot{m}_e = 16{,}725\ kg/min$$

4.

$$V_{e2} = \frac{P_1 V_{e1}}{P_2} = \frac{100 \times 72 \times 10^{-3}}{300} = 24 \times 10^{-3}\ m^3$$

$$T_4 = T_{3'} \times \left(\frac{P_4}{P_3}\right)^{\frac{n-1}{n}} = 300 \times (3)^{\frac{0,32}{1,32}} = 391,55\ K$$

$$\dot{V}_{e4} = \frac{\dot{m}_e R T_4}{P_4}$$

$$= \frac{16,72 \times 287 \times 391,55}{2,7 \times 10^6}$$

$$\dot{V}_{e4} = 0,696\ m^3/min$$

$$W_c = \frac{nP_1V_{e1}}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{1,32 \times 100 \times 10^3 \times 72 \times 10^{-3}}{0,32}\left[(3)^{\frac{0,32}{1,32}} - 1\right]$$

$$W_c = 9,063\ kJ$$

5.

$$W_i = P_1V_{e1}\ln\left(\frac{P_2}{P_1}\right)$$

$$= 100 \times 10^3 \times 72 \times 10^{-3} \times \ln(3)$$

$$W_i = 7,91\ kJ$$

6.

$$\eta_i = \frac{P_i}{P_c} = \frac{7,91}{9,063} = 0,873 = 87,274\%$$

Example 2.8

A two-stage single acting reciprocating compressor is designed for maximum efficiency. Inlet conditions are 95 kPa and 27°C. The heat loss during compression in the first stage is 1,35 kJ. The delivery pressure is 1,52 MPa. The index for compression and expansion is 1,3 for all the stages. The crank rotates at 240 rpm. The volumetric efficiency for each stage is 95%. The mechanical efficiency of the drive is 86%. The gas constant R is 287 J/kgK and C_p is 1,005 kJ/kgK.

Calculate the following:

1. pressure in the intercooler
2. final absolute temperature after compression
3. mass rate delivered
4. heat absorbed by the intercooler per second
5. motor power to drive the compressor
6. heat transfer in the two water jackets per second
7. volume rate delivered
8. stroke volume of the second stage in litres

Solutions

1.

$$r_p = \sqrt{\frac{P_3}{P_1}} = \sqrt{\frac{1{,}52 \times 10^6}{95 \times 10^3}} = 4$$

$$P_2 = 4 \times P_1 = 4 \times 95 = 380 \text{ kPa}$$

2.

$$T_2 = T_1 \times \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = 300 \times (4)^{\frac{0{,}3}{1{,}3}} = 413{,}103 \text{ K}$$

$$T_2 = T_3 = T_4 = 413{,}103 \text{ K}$$

3.

$$Q_{wj} = m_e \left(\frac{C_p - nC_v}{n-1}\right)(T_2 - T_1)$$

$$1{,}35 = m_e \left(\frac{1{,}005 - 1{,}3 \times 0{,}718}{0{,}3}\right)(413{,}103 - 300)$$

$$m_e = 50 \times 10^{-3} \text{ kg}$$

$$\dot{m}_e = m_e \times NE = 50 \times 10^{-3} \times 4 \times 1 = 0{,}2 \text{ kg/sec}$$

4.

$$Q_{ic} = \dot{m}_e C_p (T_2 - T_{2'})$$
$$= 0{,}2 \times 1{,}005(413{,}103 - 300)$$
$$Q_{ic} = 22{,}734 \text{ kJ/s}$$

5.

$$\dot{V}_{e1} = \frac{\dot{m}_e R T_1}{P_1} = \frac{0{,}2 \times 287 \times 300}{95 \times 10^3} = 0{,}181 \text{ m}^3/\text{s}$$

$$P_c = \frac{n P_1 \dot{V}_{e1}}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$
$$= \frac{1{,}3 \times 95 \times 10^3 \times 0{,}181}{0{,}3}\left[(4)^{\frac{0{,}3}{1{,}3}} - 1\right]$$
$$P_c = 28{,}092 \text{ kW}$$

$$P_{ct} = xP_c = 2 \times 28{,}092 = 56{,}183 \text{ kW}$$

$$P_m = \frac{P_{ct}}{\eta_m} = \frac{56{,}183}{0{,}86} = 65{,}329 \text{ kW}$$

6.

$$Q_{wj} = x \times Q_{wj} \times NE = 2 \times 1{,}35 \times 4 = 10{,}8 \text{ kJ/s}$$

7.

$$\dot{V}_{e3} = \frac{\dot{m}_e R T_3}{P_3}$$
$$= \frac{0{,}2 \times 287 \times 413{,}103}{1{,}52 \times 10^6}$$
$$\dot{V}_{e3} = 0{,}016 \text{ m}^3/\text{sec}$$

8.

$$\dot{V}_{e2} = \frac{\dot{m}_e R T_{2'}}{P_2}$$

$$= \frac{0{,}2 \times 287 \times 300}{380 \times 10^3}$$

$$\dot{V}_{e2} = 45{,}316 \times 10^{-3}\ m^3/sec$$

$$V_{e2} = \frac{\dot{V}_{e2}}{NE} = \frac{45{,}316 \times 10^{-3}}{4 \times 1} = 11{,}329 \times 10^{-3}\ m^3$$

$$V_{s2} = \frac{V_{e2}}{\eta_{v2}}$$

$$= \frac{11{,}329 \times 10^{-3}}{0{,}95}$$

$$V_{s2} = 11{,}925 \times 10^{-3}\ m^3 = 11{,}925\ \ell$$

Example 2.9

A two-stage single acting reciprocating compressor has a clearance volume of 2 485 cm^3in the low pressure cylinder. The pressure and temperature at the end of the suction stroke of the low pressure cylinder are 100 kPa and 22°C. The final pressure is 2,025 MPa. The stroke length is 1,2 times the diameter of the low pressure cylinder. The clearance volume is 5% of the stroke volume. The index for compression and expansion is 1,32 for both stages. The compressor is designed for maximum efficiency. The crank rotates at 240 rpm. The mechanical efficiency is 85%. The gas constant R is 287 J/kgK.

Calculate the following:

1. pressure in the intercooler
2. volumetric efficiency at inlet
3. stroke-, cylinder- and effective volume for the low pressure cylinder
4. volumetric efficiency at NTP conditions
5. diameter and stroke length of the low pressure cylinder
6. power to drive the compressor

Solutions

1.

$$r_p = \sqrt{\frac{P_3}{P_1}} = \sqrt{\frac{2{,}025 \times 10^6}{100 \times 10^3}} = 4{,}5$$

$$P_2 = 4 \times P_1 = 4{,}5 \times 100 = 450 \text{ kPa}$$

2.

$$\eta_v = 1 - \frac{V_c}{V_s}\left[\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} - 1\right]$$

$$= 1 - 0{,}05\left[(4{,}5)^{\frac{1}{1{,}32}} - 1\right]$$

$$\eta_v = 0{,}894 = 89{,}375\%$$

3.

$$V_s = \frac{V_c}{0{,}05} = \frac{2\,485 \times 10^{-6}}{0{,}05} = 49{,}7 \times 10^{-3} \text{ m}^3$$

$$V_{cyl} = V_s + V_c = 52{,}185 \times 10^{-3} \text{ m}^3$$

$$V_{e1} = V_s \times \eta_v = 49{,}7 \times 10^{-3} \times 0{,}894 = 44{,}419 \times 10^{-3} \text{ m}^3$$

4.

$$V_a = \frac{P_1 V_e T_a}{P_a T_1}$$

$$= \frac{100 \times 44{,}419 \times 288}{101{,}325 \times 295}$$

$$V_a = 42{,}798 \times 10^{-3} \text{ m}^3$$

$$\eta_{va} = \frac{V_a}{V_s} = \frac{42{,}798}{49{,}7} = 0{,}861 = 86{,}113\%$$

5.

$$V_s = LA$$

$$49{,}7 \times 10^{-3} = \frac{1{,}2D \times \pi \times D^2}{4}$$

$$D = 375 \text{ mm}$$

$$L = 450 \text{ mm}$$

6.

$$\dot{V}_{e1} = V_{e1} \times NE = 44{,}419 \times 10^{-3} \times 4 = 0{,}178 \text{ m}^3/\text{s}$$

$$P_c = \frac{nP_1\dot{V}_{e1}}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{1{,}32 \times 100 \times 10^3 \times 0{,}178}{0{,}32}\left[(4{,}5)^{\frac{0{,}32}{1{,}32}} - 1\right]$$

$$P_c = 32{,}246 \text{ kW}$$

$$P_{ct} = xP_c = 2 \times 32{,}246 = 64{,}493 \text{ kW}$$

$$P_m = \frac{P_{ct}}{\eta_m} = \frac{64{,}493}{0{,}85} = 75{,}874 \text{ kW}$$

Example 2.10

A double-acting two-stage reciprocating compressor delivers a mass rate of 3 kg/min at a pressure of 1,5 MPa. The intake conditions are 98 kPa and 28°C. The index for compression and expansion is 1,3 for all the stages. The volumetric efficiencies are 92% for the low pressure cylinder and 90% for the high pressure cylinder. The crank rotates at 240 rpm. Intercooling is incomplete and the temperature after intercooling is 80°C. The pressure in the intercooler is 360 kPa. The gas constant R is 287 J/kgK and C_p is 1,005 kJ/kgK.

Calculate the following:

1. effective volume rate for each stage
2. stroke volume for each stage
3. power of compressor
4. heat transfer to the water jackets

Solutions

1.

$$\dot{V}_{e1} = \frac{\dot{m}_e R T_1}{P_1}$$

$$= \frac{0{,}05 \times 287 \times 301}{98 \times 10^3}$$

$$\dot{V}_{e1} = 0{,}044 \text{ m}^3/\text{sec}$$

$$\dot{V}_{e2} = \frac{\dot{m}_e R T_{2'}}{P_2}$$

$$= \frac{0{,}05 \times 287 \times 353}{360 \times 10^3}$$

$$\dot{V}_{e2} = 0{,}014 \text{ m}^3/\text{sec}$$

2.

$$V_{s1} = \frac{\dot{V}_{e1}}{\eta_{v1} \times NE} = \frac{0{,}044}{0{,}92 \times 4 \times 2} = 5{,}988 \times 10^{-3} \text{ m}^3$$

$$V_{s2} = \frac{\dot{V}_{e2}}{\eta_{v2} \times NE} = \frac{0{,}014}{0{,}9 \times 4 \times 2} = 1{,}954 \times 10^{-3} \text{ m}^3$$

3.

$$P_{s1} = \frac{n P_1 \dot{V}_{e1}}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{1{,}3 \times 98 \times 10^3 \times 0{,}044}{0{,}3}\left[\left(\frac{360}{98}\right)^{\frac{0{,}3}{1{,}3}} - 1\right]$$

$$P_{s1} = 6{,}555\ \text{kW}$$

$$P_{s2} = \frac{nP_1\dot{V}_{e2}}{n-1}\left[\left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{1{,}3 \times 360 \times 10^3 \times 0{,}014}{0{,}3}\left[\left(\frac{1500}{360}\right)^{\frac{0{,}3}{1{,}3}} - 1\right]$$

$$P_{s2} = 8{,}562\ \text{kW}$$

$$P_c = P_{s1} + P_{s2} = 6{,}555 + 8{,}562 = 15{,}117\ \text{kW}$$

4.

$$T_2 = T_1 \times \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = 301 \times \left(\frac{360}{98}\right)^{\frac{0{,}3}{1{,}3}} = 406{,}414\ \text{K}$$

$$Q_{wj1} = \dot{m}_e\left(\frac{C_p - nC_v}{n-1}\right)(T_2 - T_1)$$

$$= 0{,}05\left(\frac{1{,}005 - 1{,}3 \times 0{,}718}{0{,}3}\right)(406{,}414 - 301)$$

$$Q_{wj1} = 1{,}258\ \text{kJ/s}$$

$$T_3 = T_{2'} \times \left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}} = 360 \times \left(\frac{1500}{360}\right)^{\frac{0{,}3}{1{,}3}} = 500{,}415\ \text{K}$$

$$Q_{wj2} = \dot{m}_e\left(\frac{C_p - nC_v}{n-1}\right)(T_3 - T_{2'})$$

$$= 0{,}05\left(\frac{1{,}005 - 1{,}3 \times 0{,}718}{0{,}3}\right)(500{,}415 - 360)$$

$$Q_{wj2} = 1{,}676\ \text{kJ/s}$$

Exercise 2.2

1. A three-stage reciprocating air compressor compresses 0,3 m^3 of air per
second, from an intake pressure of 90 kPa and temperature of 22 °C, to a
pressure of 2 MPa.
Intercooling is complete, and the stage pressures are in geometric progression
for the maximum efficiency.
R for air = 0,287 kJ/kg.K; C_p = 1,005 kJ/kg.K; compression index n = 1,3.

1.1 Calculate the power required to drive the compressor if a mechanical efficiency of 90% is assumed.
1.2 Calculate the heat transfer in the two intercoolers per second.
1.3 Calculate the heat transfer to the water cooling of the three cylinder jackets per second.
1.4 Calculate the isothermal efficiency.

2. A two-stage reciprocating air compressor is single-acting and has complete
intercooling and pressure ratios are equal.
The compressor delivers 12 kg/min. of air at 1,52 MPa.
Assume that intake conditions are 95 kPa and 15 °C, and that compression and expansion are polytropic with index n = 1,3.
Assume γ = 1,4 and R = 0,287 kJ/kg.

2.1 Calculate the power required.
2.2 Calculate the isothermal efficiency.
2.3 Calculate the volume of free air delivered per minute.
2.4 Calculate the heat transferred in each cylinder and the heat transferred in the intercooler per second. (Q = 11,42 kJ/s)

2.5 Calculate the swept and clearance volumes for the LP cylinder if the speed is 450 r/min. and clearance ratio for the LP cylinder is 4%.

3. A two-stage, double-acting reciprocating compressor runs at 300 r/min.
Its low pressure cylinder has a diameter of 250 mm and a stroke of 183,3 mm.
At the end of the low-pressure suction stroke, the air in the cylinder has a
pressure of 94,5 kPa and a temperature of 27 °C.

The pressure at final delivery is 500 kPa. Intercooling is complete and the
stage pressures are in geometric progression.
The clearance volume in the low pressure cylinder is 450 ml.

3.1 Calculate the volumetric efficiency of the low pressure cylinder if the index of compression is 1,3.
3.2 Calculate the mass of air compressed per minute.
3.3 Calculate the total power required for compression.
3.4 Calculate the heat transferred in the intercooler if the specific heat capacity of air at constant pressure is 1,005 kJ/kg.K.

Module summary

Formulae

$$\frac{P_2}{P_1} = \frac{P_3}{P_2} = r_p = \sqrt[x]{\frac{P_{x+1}}{P_1}}$$

$$m_e R = \frac{P_1 V_{e1}}{T_1} = \frac{P_2 V_{e2}}{T_{2'}} = \frac{P_3 V_{e3}}{T_{3'}} = \frac{P_a V_a}{T_a}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

$$\eta_{v1} = 1 - \frac{V_c}{V_s}\left[\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} - 1\right]$$

or

$$\eta_{v1} = \frac{V_e}{V_s}$$

$$\eta_{va} = \frac{V_a}{V_s}$$

$$V_{cyl} = V_s + V_c$$

$$Q_{ic} = m_e C_p (T_2 - T_{2'})$$

$$Q_{wj} = W_c - Q_{ic}$$

or

$$Q_{wj} = m_e \left(\frac{C_p - nC_v}{n-1}\right)(T_2 - T_1)$$

$$W_c = \frac{nmRT_1}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

or

$$W_c = \frac{nP_1V_{e1}}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$P_c = \frac{W_c}{NE}$$

$$P_c = W_c \times NE$$

Summative assessment: Module 2

1. A two-stage reciprocating air compressor takes in air at 95 kPa and
delivers it at the intercooler at 750 kPa, and then increases the pressure
to 6 MPa in the second stage.

- The compression index is 1,35.
- The stroke volume of the low pressure cylinder is 400 ml.
- The volumetric efficiency for both cylinders is 87,5%.

1.1 Calculate the work required per cycle if it rotates at 360 r/min and the mechanical efficiency is 82%.
1.2 Calculate the mass of air delivered per hour if atmospheric conditions remain the same at 95 kPa and 17 °C when drawn into the low pressure cylinder.

2. A three-stage reciprocating compressor compresses 10 m3 of air every minute, from 96 kPa and 17 °C to 2 592 kPa.
The stage pressures are in geometric progression and after each compression
the air is cooled back to 17 °C. Assume that CP = 1,005 kJ/kg.K and that
n = 1,35.

2.1 Calculate the power to drive the compressor if the mechanical efficiency is 91%.
2.2 Calculate the heat transferred in each intercooler and the aftercooler per second.
2.3Calculate the heat transferred through the three cylinder walls per second.
2.4 Calculate the isothermal work and isothermal efficiency.

3. A three-stage compressor delivers air to a pressure of 2 MPa from atmospheric pressure of 103 kPa.
The temperature at the start of compression is 30 °C, and the index of compression in each stage is 1,3. Minimum work conditions prevail.

3.1 Determine the intermediate pressures.
3.2 Determine the temperatures at the end of compression in each stage.
3.3 Determine the heat extracted per kg of air in each intercooler.
3.4 Determine the work done per kg of air delivered.
3.5 Determine the isothermal work done per kg of air if the compression were isothermal.
3.6 Draw the complete cycle on a P–V diagram, neglecting clearance volume.
For air take adiabatic index $\gamma = 1,4$ and $R = 0,287$ kJ/kg.K.

4. Do the following.
4.1 Draw a pressure–volume diagram of a two-stage reciprocating air
compressor with a clearance volume
4.2 Briefly explain the effect of clearance on the volumetric efficiency of the
machine.

5. A two-stage air compressor working at minimum power conditions compresses 10,8 m3/min. of free air to a pressure of 1,25 MPa. Suction conditions are 90 kPa and 29 °C.

5.1 Determine the power required to drive the compressor if the mechanical efficiency is 80%.
5.2 Determine the heat extracted in the cooler between stages. Neglect clearance, take the index of compression as 1,2 and free air conditions as 101 kPa and 15 °C.
For air R = 0,287 kJ/kg.K and CP = 1,005 kJ/kg.K

MODULE

3 *Nozzles*

By the end of this module, students should be able to:

- analyse given data to determine the inlet-, throat- and exit conditions;
- consider adiabatic conditions at the throat and exit;
- apply Mach number in calculations;
- analyse given data to determine the type of steam and the specific enthalpy by calculation or use of steam tables at the inlet, throat and exit;
- apply data in calculations at the inlet-, throat- and exit points; and
- consider the actual and isentropic conditions at the throat and exit.

Introduction

Many applications (steam and gas turbines, jet engines, rocket motors, flow measurements, and so on) require a high velocity stream of fluid, and the nozzle is the best way to obtain this. When a fluid is decelerated in a duct and causes a rise in pressure along the stream, the duct is called a diffuser. Two practical applications of diffusers are the centrifugal compressor and the ram jet.

3.1 Gas nozzles

A nozzle is a duct of smoothly varying cross-sectional area where a steadily-flowing fluid can be made to accelerate by a pressure drop along the duct. Many applications (steam and gas turbines, jet engines, rocket motors, flow measurements, and so on) require a high velocity stream of fluid, and the nozzle is the best way to obtain this. When a fluid is decelerated in a duct and causes a rise in pressure along the stream, the duct is called a **diffuser**. Two practical applications of diffusers are the centrifugal compressor and the ram jet.

For the purposes of this section, nozzles will be considered as fitted to a gas turbine engine and assumed air to be the working substance. Allowance will be made for instances where a perfect gas is used as the working substance instead of air. When air is used, the adiabatic index γ will be assumed to be 1,4.

When the gas leaves the combustion chamber of a gas turbine, as much as possible of all the forms of energy that the gas stream possesses must be converted into kinetic energy for the gas to impinge on the turbine blades at high speeds.
For this to happen, a row of nozzles is fitted between the combustion chamber outlet and the turbine blades.

Nozzle shape

The shape of the nozzle should allow the conversion from internal energy to kinetic energy efficiently. Two types of nozzle, convergent and convergent-divergent nozzle, are used. The selection of a nozzle depends on the pressure drop that occurs between inlet and outlet.

It is not possible to examine the design of nozzles in detail in this book. However, more advanced work shows that the area of cross-section of a nozzle decreases to a minimum at a pressure equal to about half the inlet pressure. This minimum section of the nozzle is called the **throat** and the corresponding pressure at the throat is the **critical** pressure. The type of nozzle required depends on the discharge pressure. If the discharge pressure is greater than the critical pressure, a convergent nozzle is required. If the discharge pressure is less than the critical pressure, a convergent divergent nozzle has to be used.

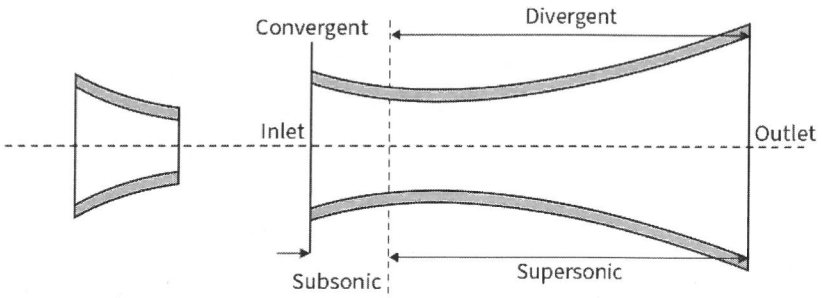


Figure 3.1

The shape of the convergent-divergent nozzle is usually determined by calculating the throat and exit areas, and making a simple conical

section with a 12° angle of divergence. The inlet to the throat is a smooth curve with a short length.

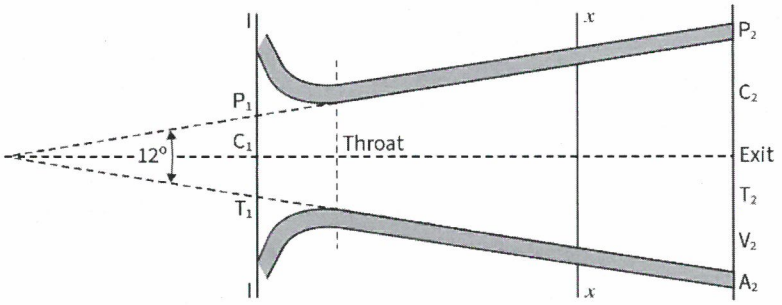


Figure 3.2

The velocity at the throat of a nozzle operating at its designed pressure ratio, is the velocity of sound at the throat conditions. The flow up to the throat is **subsonic**, and the flow after the throat is **supersonic**. A sonic or a supersonic flow requires a diverging duct to accelerate it.

The specific volume of a liquid is constant over a wide pressure range, therefore nozzles for liquids are always convergent, even at very high exit velocities (for example, a fire hose uses a convergent nozzle), as shown in the diagram.

Consider a stream of fluid at pressure P1, enthalpy (total heat) $h1$ and a low velocity C1 at the entrance of the nozzle. The shape of duct that will cause the fluid (gas or air) to accelerate to a high velocity as the pressure falls along the duct has to be found. It can be assumed that the heat loss from the duct is negligible (for example, adiabatic flow Q = 0). It is clear that no work is done on or by the fluid (for example, W = 0). Between section 1 and any other section (for example, X–X), where the pressure is P_2, the enthalpy is h_2 and the velocity is C_2.

Apply the steady-flow formula.

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

$$C_2^2 = 2(h_1 - h_2) + C_1^2$$

$$C_2 = \sqrt{2(h_1 - h_2) + C_1^2}$$

.....(1)

However, for a perfect gas, enthalpy (total heat): $h = C_p \times T$

h_1 becomes C_pT_1 and h_2 becomes C_pT_2

$$C_2 = \sqrt{2(C_pT_1 - C_pT_2) + C_1^2}$$

$$C_2 = \sqrt{2C_p(T_1 - T_2) + C_1^2}$$

.....(2)

If the area at section X-X is A_2 and rthe specific volume is V_2:

Mass (m) of flow $= \frac{C_2A_2}{V_2}$ or area per unti mass flow $\frac{A_2}{m} = V_2/$

C_2.....(3)

Substituting for velocity C_2 in, for example:

Area per unit mass flow $= \frac{A_2}{m}$

$$= \frac{V_2}{\sqrt{2C_p(T_1-T_2)+C_1^2}}$$

.....(4)

From equation 4 it can be seen that, to find the way in which the area of the duct varies, it is necessary to be able to evaluate the specific volume V, and the total heat (enthalpy) C_pT, at any section X–X.

To do this, some information about the process between section 1 and section X–X must be known. For the ideal frictionless case, since

the flow is adiabatic and reversible, the process is adiabatic (isentropic).

S_1 = entropy at any section X–X = $S_0 2$

Using formula 3 and the fact that S1 = S2, it is possible to plot the variation of the cross-sectional area of the duct against the pressure along the duct. For a vapour, this can be done by using tables. For a perfect gas the procedure is simpler, since PV^{γ} = constant for an adiabatic (isentropic) process.

Critical pressure ratio

The velocity at the throat of a correctly designed nozzle is the velocity of sound. Similarly, for a nozzle that is converged only, the fluid will attain sonic velocity at exit if the pressure drop across the nozzle is large enough. The ratio of the pressure at the section where sonic velocity is attained to the inlet pressure of a nozzle, is called the critical pressure ratio.

In most practical applications, the velocity of the inlet to the nozzle is negligible compared to the exit velocity, and from (for example) ②:

$C_2 = \sqrt{(2C_p(T_1 - T_2) + C_1^2}$

And, neglecting C_1^2 (inlet velocity):

$C_2 = \sqrt{2C_p(T_1 - T_2)}$ (C_2 = velocity of flow in m/s at X-X).....⑤

C_p = specific heat capacity of gas/air at constant pressure in J/kg.K

Substitutng into (for example) ③:

Area per unit mass flowL $\frac{A_2}{m} = \frac{V_2}{C_2}$; where m is 1 kg gas

$= \frac{V_2}{\sqrt{2C_p(T_1-T_2)}} m^3$

∴ area per kg of gas $A_2 =$

$\frac{V_2}{\sqrt{2C_p(T_1-T_2)}} m^3$ .....⑥

From equation ⑥, the critical pressure ratio at the throat:

$\frac{P_C}{P_1} = \frac{2}{\gamma+1}^{\frac{\gamma}{\gamma-1}}$

.....⑦

The ratio of the temperature at the section of the nozzle where velocity is sonic to the inlet temperature, is called the critical temperature ratio.

Critical temperature ratio $\frac{T_C}{T_1} = \left(\frac{P_C}{P_1}\right)^{\frac{\gamma}{\gamma-1}}$

$= \frac{2}{\gamma+1}$

.....

⑧

Also, the critical velocity of a correctly designed convergent-divergent nozzle is the velocity at the throat of the convergent-divergent nozzle:

Critical velocity $= C_c$

$= \sqrt{2C_p(T_1 - T_C)}$

m/s.....⑨

Area per kg of gas $= A_C$

$= \sqrt{2C_p(T_1 - T_C)}$

m^2 ..... (10)

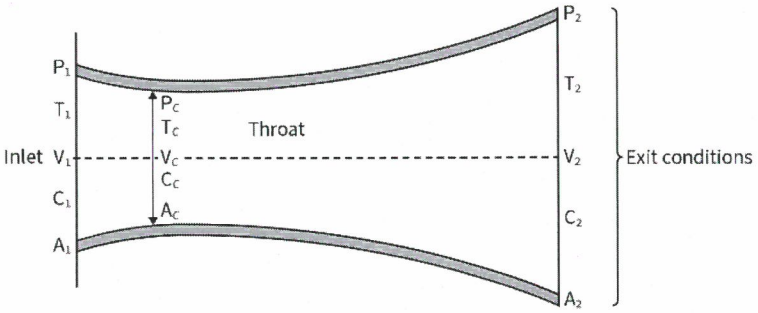


Figure 3.3

Let *c* represent the critical conditions at the throat, where it can be proved:

- $P_c = P_1\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$ P_c = critical pressure in kPa
- $T_c = T_1\left(\frac{2}{\gamma+1}\right)$ K T_c = critical temperature Kelvin
- $V_c = \frac{RT_c}{P_C}$ m^3 V_c = critical volume in m^3/kg and m = 1 kg
- $C_c = \sqrt{2C_p(T_1 - T_C)}$ m/sC_c – critical velocity in m/s
- $A_c = \frac{mV_c}{C_c}$ m^2 m = mass of air in kg

= area of throat in m^2

Let 2 represent the exit conditions:

- $T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$ Kelvin
- $V_2 = \frac{RT_2}{P_2}$ m^3/kg m =1 kg
- $C_2 = \sqrt{2C_p(T_1 - T_2)}$ m/s

- $A_2 = \frac{mV_2}{C_2} m^2$

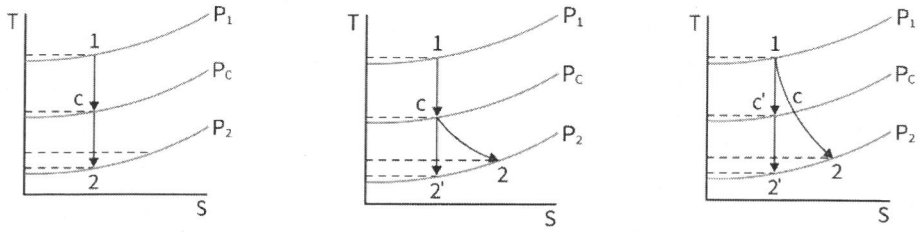


Figure 3.4 Friction in nozzle

Figure 3.4(a) show adiabatic expansion, where nozzle efficiency is 100%.

In Figure 3.4(b), there is friction in the diverging part only.

$$T_{2'} = T_c \times \left(\frac{P_2}{P_c}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = T_c \times \left(\frac{P_2}{P_c}\right)^{\frac{n-1}{n}}$$

Figure 3.4(c) Shows friction in diverging and converging parts

Path 1-c'-2' adiabatic expansion (ideal)

Path 1-c-2 polytropic expansion (actual)

$$T_{2'} = T_1 \times \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = T_1 \times \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

$$T_c = T_1 \times \left(\frac{P_c}{P_1}\right)^{\frac{n-1}{n}}$$

$$\eta_c = \frac{T_1 - T_c}{T_1 - T_{c'}} \text{ (converging efficiency)}$$

$$\eta_d = \frac{T_c - T_2}{T_c - T_{2'}} \text{ (diverging efficiency)}$$

$$\eta_n = \frac{T_1 - T_2}{T_1 - T_{2'}} \text{ (nozzle efficiency)}$$

Example 3.1

Air at 860 kPa and 190 °C expands at the rate of 4,5 kg at 103 kPa through a convergent-divergent nozzle into a space. Gamma = 1,4. R for air = 0,287 kJ/kg.K and C_p = 1,005 kJ/kg.K. Assuming that the inlet velocity is negligible, calculate the throat and exit areas of the nozzle.

To find the area at the throat (A_c), find the critical press (P_c).

Solution

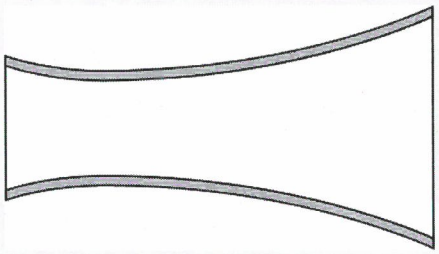


Figure 3.5

$P_1 = 860$ kPa
$T_1 = 190 + 273 = 463$ K
$R = 0{,}287$ kJ/kg.K
$m = 4{,}5$ kg

$P_2 = 103$ kPa
$C_p = 1{,}005$ kJ/kg.K
$C_1 = 0$
$\gamma = 1{,}4$

$$P_c = P_1 \left(\frac{2}{y+1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 860 \left(\frac{2}{1,4+1}\right)^{\frac{1,4}{1,4-1}}$$
$$= 454 \text{ kPa}$$

Note

If gamma = 1,4 in every problem, then $\frac{P_c}{P_1} = 0,525$

$$\therefore P_c = P_1 \times 0,528$$
$$= 860 \times 0,528$$
$$= 454 \text{ kPa}$$

Find the critical temperature (T_c)

$$T_c = T_1 \left(\frac{2}{y+1}\right)$$
$$= 463 \times \frac{2}{2,4}$$
$$= 385,8 \text{ K}$$

Find the critical temperature (V_c)

$$V_c = \frac{RT_c}{P_c}$$
$$= \frac{0,287 \times 385,8}{454}$$
$$= 0,244 \text{ m}^3 \text{ per kg}$$

Note

m is always 1 in this formula, as we are calculating the specific volume m^3/kg.

Find the critical velocity

$$C_c = \sqrt{2C_p(T_1 - T_c)}$$
$$= \sqrt{2 \times 1,005 \times 10^3 (463 - 385,8)}$$
$$= \sqrt{2 \times 1,005 \times 77,5}$$
$$= \sqrt{155\,000}$$

$= 394$ m/s

Find the throat area (A_c)

$$A_c = \frac{m \times V_c}{C_c}$$
$$= \frac{4{,}5 \times 0{,}244}{394}$$
$$= 0{,}00279 \text{ m}^2$$
$$= 2\,790 \text{ mm}^2$$

To find the exit area A_2, first find the exit temperature (T_2)

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{y-1}{y}}$$
$$= 463 \left(\frac{103}{860}\right)^{\frac{0{,}4}{1{,}4}}$$
$$= 252 \text{ K}$$

Find the exit volume (V_2)

$$V_2 = \frac{mRT_2}{P_2}$$
$$= \frac{1 \times 0{,}287 \times 252}{103}$$
$$= 0{,}703\ m^3 \text{ per kg}$$

Find the velocity at the exit (C_2)

$$C_2 = \sqrt{2C_p(T_1 - T_2)}$$
$$= \sqrt{2 \times 1\,005(463 - 252)}$$
$$= 651 \text{ m/s}$$

Note

C_p must always be joules/kg.K.

Find the exit area (A_2)

$$A_2 = \frac{m \times V_2}{C_2}$$

$$= \frac{4,5 \times 0,702}{651}$$
$$= 0,00485 \text{ mm}^2$$
$$= 4\,850 \text{ mm}^2$$

Example 3.2

A convergent-divergent nozzle expands air at 689 kPa and 427 °C to 100 kPa. The throat area of the nozzle = 650 mm2 and R for air = 287 J/kg.K. Assume gamma = 1,4 and calculate the mass of air flow through the nozzle in kilogram per second, and inlet velocity is negligible and C_p is 1,005 kJ/kg.K.

Solution

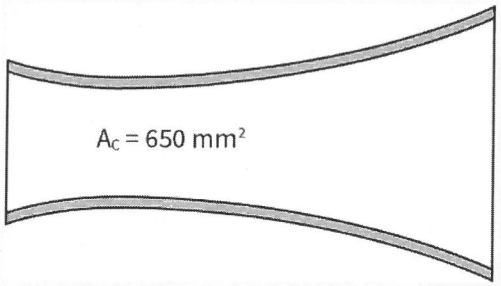


Figure 3.6

$P_1 = 689 \text{ kPa}$
$P_2 = 100 \text{ kPa}$
$R = 0,287 \text{ kJ/kg.K}$
$T_1 = 427 + 273 = 700 \text{ K}$
$\gamma = 1,4$
$C_1 = 0$
$C_p = 1,005 \text{ kJ/kg.K}$

Find the mass (m) of air flow

$$P_c = P_1 \left(\frac{2}{y+1}\right)^{\frac{y-1}{y}}$$
$$= 689 \left(\frac{2}{2,4}\right)^{\frac{1,4}{0,4}}$$

$= 364$ kPa

$$T_c = T_1\left(\frac{2}{y+1}\right)$$
$$= 700 \times \frac{2}{2,4}$$
$$= 583 \text{ K}$$

$$C_c = \sqrt{2C_p(T_1 - T_c)}$$
$$= \sqrt{2 \times 1\,005(700 - 583)}$$
$$= 485 \text{ m/s}$$

$$V_c = \frac{mRT_c}{P_c}$$
$$= \frac{1 \times 0,287 \times 583}{364}$$
$$= 0,458 \text{ m}^3 \text{ per kg}$$

From $A_c = \frac{m \times V_c}{C_c}$:

$$m = \frac{A_c \times C_c}{V_c}$$
$$= \frac{650}{10^6} \times \frac{485}{0,458}$$
$$= 0,688 \text{ kg/s}$$

Example 3.3

A gas in a convergent-divergent nozzle expands from 500 kPa to 143 kPa. The initial temperature is 537 °C, and the adiabatic efficiency of the nozzle is 92%. Assume that all losses take place after the throat of the nozzle and that gamma is 1,4. R = 288 J/kg.K and C_p 1 008 J/kg.K.

Calculate the throat and exit areas of the nozzle per kilogram of gas per second.

Solution

$P_1 = 500$ kPa
$m = 1$ kg/s
$\gamma = 1{,}4$
$C_1 = 0$
$P_2 = 143$ kPa
$T_1 = 537 + 273 = 810$ K
$R = 288$ kJ/kg.K
$C_p = 1\,008$ kJ/kg.K

Find the throat area (A_c)

$$P_c = P_1\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma-1}{\gamma}}$$
$$= P_1 \times 0{,}528$$
$$= 500 \times 0{,}528$$
$$= 264 \text{ kPa}$$

$$T_c = T_1\left(\frac{2}{\gamma+1}\right)$$
$$= 700 \times \frac{T_1}{1{,}2}$$
$$= \frac{810}{1{,}2}$$
$$= 675 \text{ K}$$

$$V_c = \frac{RT_c}{P_c}$$
$$= \frac{0{,}288 \times 675}{264}$$
$$= 0{,}739 \text{ m}^3 \text{ per kg}$$

$$C_c = \sqrt{2C_p(T_1 - T_c)}$$
$$= \sqrt{2 \times 1\,008(810 - 675)}$$
$$= 522 \text{ m/s}$$

$$A_c = \frac{m \times V_c}{C_c}$$
$$= \frac{1 \times 0{,}739}{522}$$
$$= 0{,}00142 \text{ m}^2$$

$= 1\,420\ \text{mm}^2$

Fine the area at exit (A_2)

$$T_{2'} = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$
$$= 810\left(\frac{143}{500}\right)^{\frac{0,4}{1,4}}$$
$$= 566\ \text{K}$$

This is the adiabatic temperature at exit and the nozzle has an adiabtic efficiency of 92%, so actual exit temperature T_2 must be found.

Use the expansion adiabatic efficiency formula:

$$\eta_n = \frac{T_1 - T_2}{T_1 - T_{2'}}$$
$$0,92 = \frac{810-T_2}{810-566}$$
$$T_2 = 586\ \text{K}$$

$$V_2 = \frac{RT_2}{P_2}$$
$$= \frac{0,288\times586}{143}$$
$$= 1,18\ \text{m}^3\ \text{per kg}$$

$$A_2 = \frac{m\times V_2}{C_2}$$
$$= \frac{1\times1,18}{672}$$
$$= 0,001756\ \text{m}^2$$
$$= 1\,756\ \text{mm}^2$$

$$C_2 = \sqrt{2C_p(T_1 - T_2)}$$
$$= \sqrt{2\times1\,008(810-586)}$$

$= 672$ m/s

Example 3.4

Air at 860 kPa and 190 °C expands adiabatically at the rate at 5 kg per second, through a convergent-divergent nozzle into a space of 103 kPa. The critical temperature at the throat is 386 K. Assume the inlet velocity is negligible and R for air is 288 J/kg.K.

1. Calculate the value of gamma.
2. Calculate the value of C_p.
3. Calculate the throat and exit areas of the nozzles.

Solutions

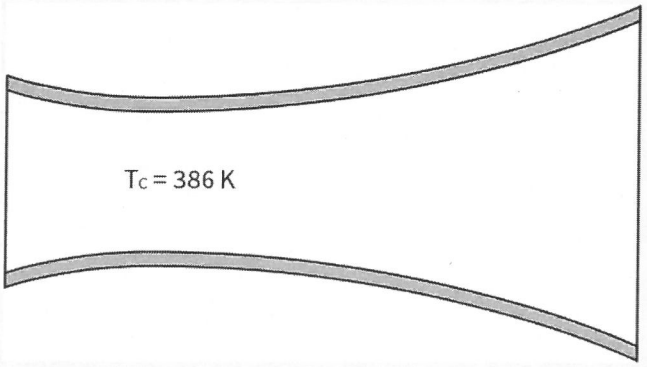


Figure 3.7

$T_1 = 190 + 273 = 463$ K
$P_1 = 860$ kPa
$C_1 = 0$
$V_1 = 0$
$P_2 = 103$ kPa
$R = 288$ kJ/kg
$T_c = T_1 \left(\frac{2}{\gamma+1}\right)$

$$386 = 463\left(\frac{2}{\gamma+1}\right)$$

$$\gamma = \frac{540}{386}$$

$$= 1{,}4$$

2.

$$R = C_p - C_V \text{ and } \gamma = \frac{C_p}{C_v}$$

$$C_V = \frac{C_p}{1{,}4}$$

$$0{,}288 = C_p - \frac{C_p}{1{,}4}$$

$$0{,}4\ C_p = 0{,}288 \times 1{,}4$$

$$C_p = \frac{0{,}288 \times 1{,}4}{0{,}4}$$

$$= 1{,}009 \text{ kJ/kg.K}$$

3. Find the throat area

$$P_c = P_1 \times 0{,}528$$

$$= 860 \times 0{,}528$$

$$= 454 \text{ kPa}$$

$$V_c = \frac{RT_c}{P_c}$$

$$= \frac{0{,}288 \times 386}{454}$$

$$= 0{,}244 \text{ m}^3 \text{ per kg}$$

$$C_c = \sqrt{2C_p(T_1 - T_c)}$$

$$= \sqrt{2 \times 1\,008(463 - 386)}$$

$$= 394 \text{ m/s}$$

$$A_c = \frac{m \times V_c}{C_c}$$

$$= \frac{5 \times 0{,}244}{394}$$

$$= 0{,}00331 \text{ m}^2 \text{ or } 3\,100 \text{ mm}^2$$

Find the exit area

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$
$$= 463 \left(\frac{103}{860}\right)^{\frac{0,4}{1,4}}$$
$$= 252 \text{ K}$$

$$V_2 = \frac{RT_2}{P_2}$$
$$= \frac{0,288 \times 252}{103}$$
$$= 0,702 \text{ m}^3 \text{ per kg}$$

$$C_2 = \sqrt{2C_p(T_1 - T_2)}$$
$$= \sqrt{2 \times 1\,008(463 - 252)}$$
$$= 652 \text{ m/s}$$

$$A_2 = \frac{m \times V_2}{C_2}$$
$$= \frac{5 \times 0,702}{652}$$
$$= 0,0054 \text{ m}^2 \text{or } 5\,400 \text{ mm}^2$$

Exercise 3.1

1. Air enters a nozzle with a pressure of 700 kPa and a temperature of 180 °C. Exit pressure = 100 kPa. The law connecting pressure and specific volume during expansion in the nozzle is $PV^{1,3}$ = constant. Determine the velocity at exit of nozzle. Take C_p = 1,006 kJ/kg.K.

2. Air enters a convergent-divergent nozzle at 3 MPa and with a temperature of 400 °C. It leaves at a pressure of 0,5 MPa. The exit area = 5 000 mm^2. Expansion through the nozzle is adiabatic

according to the law PV^{γ} = constant. Assume gamma = 1,4 and R = 287 J/kg.K.

2.1 Determine the mass of air flow through the nozzle in kg per second.
2.2 Determine the area at the throat.

3. Air enters a nozzle at a pressure of 3,5 MPa and a temperature of 500 °C. It leaves at a pressure of 0,7 MPa. The air flow rate through the nozzle = 1,3 kg per second. Expansion may be considered to be adiabatic and follows the law
PV^{γ} = constant.
Assume gamma = 1,4; R = 0,287 kJ/kg.K, and C_p = 1,005 J/kg.K.

3.1 Calculate the throat area.
3.2 Calculate the exit area.

4. A gas expands in a convergent-divergent nozzle from 800 kPa to 200 kPa. The temperature at the entrance of the nozzle = 900 °C. Assume that all losses take place after the throat of the nozzle, designed for an diverging efficiency of 90%. Gamma = 1,4; R = 0,289 kJ/kg.K, and C_p = 1,005 J/kg.K.

Calculate the following per kilogram of gas per second.
4.1 the throat area
4.2 the exit area

5. A convergent-divergent nozzle expands air at 750 kPa and 500 °C to 150 kPa. The exit area of the nozzle = 720 mm^2 and R = 288 J/kg.K; gamma = 1,4. Neglect inlet velocity.

5.1 Calculate the value of C_p.
5.2 Calculate the mass of air flow through the nozzle in kilograms per second.

6. Air enters a convergent-divergent nozzle at a pressure of 4 MPa and a temperature of 550 °C. It leaves at a pressure of 900 kPa. The air flow rate through the nozzle is 1,4 kg/s; expansion follows the law PV^{γ} = constant; gamma = 1,4; C_p = 1 005 kJ/kg.K.

6.1 Calculate the value of R.
6.2 Calculate the throat area.
6.3 Calculate the exit area.

3.2 Nozzles for steam flow

The flow of steam through a nozzle could be regarded, in its simplest form, as an adiabatic expansion. The steam enters the nozzle with a relatively small velocity and a high initial pressure. The initial velocity is so small compared to the final velocity that it may be neglected.

As the steam expands, the velocity will increase as the heat energy at the steam is converted to kinetic energy. During the expansion of the steam through the nozzle, no heat is supplied or rejected (adiabatic law) and, although no external work is done on a piston, work is done by increasing the kinetic energy of the steam. The expansion is therefore an adiabatic expansion, and the flow through the nozzle is regarded as an adiabatic flow.

As the steam loses its pressure in passing through the nozzle, it is also losing its total heat (enthalpy). The change of total heat of the steam must therefore equal the increase kinetic energy. Hence the work done is equal to the heat drop.

The expansion at steam through a nozzle is not a free expansion, and the steam is not throttled as it has a large velocity at the end of expansion. Work is done by the expanding steam in producing this kinetic energy.

In practice, friction is produced between the steam and the sides at the nozzle. This friction causes a resistance to the flow, which is converted into heat. The heat that is formed tends to dry the steam. The effect of the friction, in resisting the flow and drying the steam, should be taken into account in the design of the nozzle, as it makes an appreciable difference to the results.

Another complication in the design of nozzles is super saturation, which happens due to a time lag in the condensing of the steam during the expansion. The expansion takes place very rapidly and, if the steam is initially dry or superheated, it should become wetter as the pressure falls as the expansion is adiabatic. As a result of the rapidity of the expansion, the steam does not have time to condense, but remains in an unnatural dry or superheated state. Then, at a certain instant, it will suddenly condense to its natural state.

This type of flow is called a supersaturated flow, which will affect the mass and condition of the steam discharged.

The flow of steam through a nozzle could be one of the following:
- an adiabatic flow
- an adiabatic flow modified by friction
- a supersaturated flow

The expansion of steam through a nozzle is theoretically adiabatic. However, the adiabatic index gamma (γ) cannot be used, as $\gamma = \frac{C_p}{C_v}$ and is used only for gas.

For superheated steam, the adiabatic index as an average value of n = 1,3.

Using the equation $P_c = P_1 \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$ given in a gas nozzle:

$P_c = P_1 \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$ $\qquad n = 1,3$

$= P_1 \left(\frac{2}{1,3+1}\right)^{\frac{1,3}{0,3}}$

$= 0,546\ P_1$

This is the critical pressure ratio for superheated steam.

In the equation $C_c = \sqrt{2C_p(T_1 - T_C)}$ (gas flow) for steam, enthalpy (total heat), *h* replaces $C_p \times T$.

$C_c = \sqrt{2(h_1 - h_c)}$, where h_1 and h_c are values for enthalpy in kJ/kg found from the steam tables or from an enthalpy-entropy h-s chart, but the values of h_1 and h_c should be in joules per kg.

$C_2 = \sqrt{2(h_1 \times 10^3) - (h_c \times 10^3)}$ (velocity at throat)

$= \sqrt{(2 \times 10^3) - (h_1 \times h_c)}$

$= \sqrt{2\ 000(h_1 \times h_c)}$ m/s important

The steam tables and the enthalpy-entropy h-s chart do not give the volume of superheated steam, but the volume can be found from an empirical formula.

$V = \frac{n-1}{n}\left(\frac{h-1941}{P}\right)$

h = enthalpy of superheated steam in kJ/kg; P = pressure

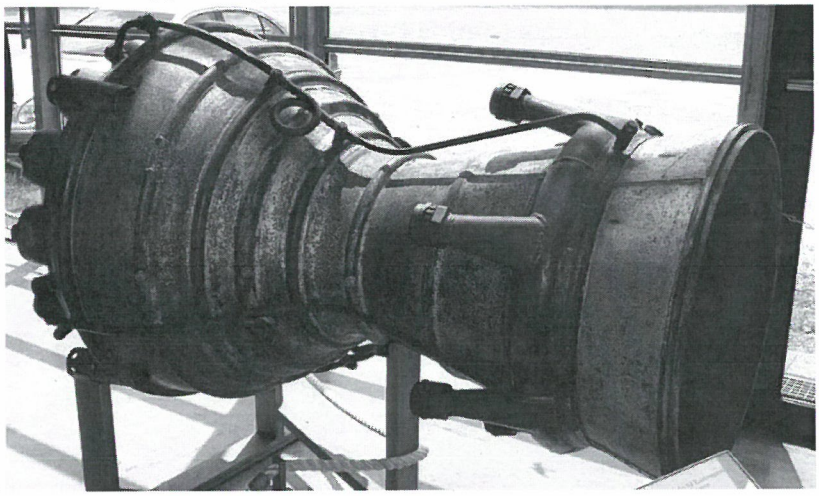


Figure 3.8: Rocket motors also employ convergent-divergent nozzles

Nozzle efficiency

In practice, expansion in the nozzle is accompanied by fluid friction, which causes internal heating of the fluid. It results in an increase in entropy and a smaller drop in enthalpy (with a consequent loss of velocity), compared to the frictionless isentropic (adiabatic) expansion 1 to 2 in the diagram. The actual expansion is shown by 1 to 2. These effects of friction are allowed for by a nozzle efficiency.

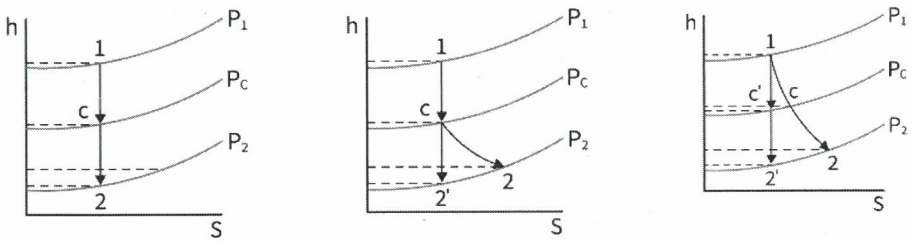


Figure 3.9: Friction in nozzle

$$\text{Nozzle efficiency} = \frac{\text{Actual enthalpy drop}}{\text{Isentropic enthalpy drop}}$$

$$\frac{h_1 - h_2}{h_1 - h_{21}}$$

Notes on figure

Figure 3.9(a): No friction, nozzle efficiency is 100%

Figure 3.9(b): Friction in diverging part only

Figure 3.9(c): Friction in diverging and converging parts

Path 1-c'-2' expansion without friction (ideal)

Path 1-c-2 is expansion with friction (actual)

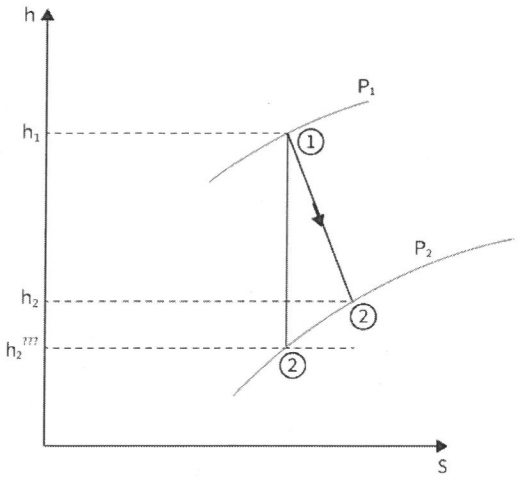


Figure 3.10

$\eta_c = \frac{h_1 - h_c}{h_1 - h_{c'}}$ (converging efficiency)

$\eta_d = \frac{h_c - h_2}{h_c - h_{2'}}$ (diverging efficiency)

$\eta_n = \frac{h_1 - h_2}{h_1 - h_{2'}}$ (nozzle efficiency)

Steam revision table

Below is a table summary of all the relevant formulae required to calculate specific enthalpy and specific volume of steam using the steam tables.

	Specific enthalpy (kJ/kg)	Specific volume (m^3/kg)
Water	h_f from tables @ t_s $\Delta h = C_p \Delta t_w$	$V = 0{,}001$
Wet steam	$h = h_f + xh_{fg}$ @ P_s	$V = xV_g$
Dry steam	$h = h_g$ @ P_s	$V = V_g$
Sup steam	$h = h_g + C_p(t_{su} - t_s)$ @ P_s h $= h_{su}$ from sup tables @ P_s	V $= \frac{(n-1)}{n}\left(\frac{h_{su} - 1\,941}{P_s}\right)$ $V = V_{su}$ @ P_s

Table 3.1

Calculations on steam nozzles

Example 1.5

Steam enters a convergent-divergent nozzle at a pressure of 2 MPa and a temperature of 250 °C, and leaves the nozzle at a pressure of 450 kPa with an isentropic dryness factor of 0,94. The steam at the throat is dry-saturated with a temperature of 184,1 °C. The steam flows through the nozzle at a rate of 330,72 kg/min.
Assume the reheat factor is 10% of the overall heat drop due to friction. Use only the steam tables to answer the questions.

1. Determine the pressure in kPa, the specific enthalpy in kJ/kg, the velocity in m/s, the area in mm^2 and the diameter in mm at the throat.

2. Determine the specific isentropic enthalpy in kJ/kg and the specific actual enthalpy in kJ/kg at the exit.
3. Determine the velocity in m/s, the actual dryness factor, the specific volume in cubic metres per kg and the area in mm^2 at the exit.

Solutions

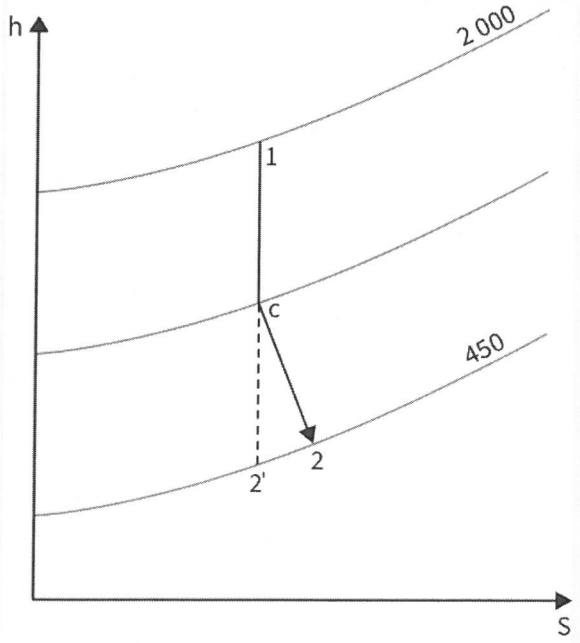


Figure 3.11

P_1 = MPa
t_{su} = 250°C
P_2 = 450 kPa
x_{2^1} = 0,94;
t_c = 184,1° C
m = 330,72 kg/min
η_n = 90%

1. From steam tables

At 184,1 °C $P_c = 1\ 100$ kPa

$h_c = h_g$

$= 2\ 780$ kJ/kg

At 2 000 kPa and 250 °C

$h_1 = h_{su}$

$= 2\ 904$ kJ/kg

$$C_c = \sqrt{2 \times 10^3 (h_1 - h_c)}$$

$$= \sqrt{2 \times 10^3 (2\ 904 - 2\ 780)}$$

$= 497{,}906$ m/s

$V_c = V_g$

$= 0{,}1774\ m^3$ kg at 1 100 kPa

$$A_c = \frac{mV_c}{C_c}$$

$$= \frac{33072 \times 0{,}1774}{60 \times 497{,}906}$$

$= 1\ 963{,}527\ mm^2$

$$d_c = \sqrt{\frac{A_c}{\pi}}$$

$$= \sqrt{\frac{4 \times 1\ 963{,}527}{\pi}}$$

$= 50$ mm

2.

$h_{2'} = h_f + x_{2'} \times h_{fg}$ at 450 kPa

$= 624 + 0{,}94 \times 2\ 120$

$= 2\ 616{,}8$ kJ/kg

$$\eta_n = \frac{h_1 - h_2}{h_1 - h_{2'}}$$

$$0{,}94 = \frac{2\,904 - h_2}{2\,904 - 2\,616{,}8}$$

$$h_2 = 2\,645{,}52 \text{ kJ/kg}$$

3.

$$C_2 = \sqrt{2 \times 10^3 (h_1 - h_2)}$$

$$= \sqrt{2 \times 10^3 (2\,904 - 2\,645{,}52)}$$

$$C_c = 719 \text{ m/s}$$

$$h_2 = h_f + x_2 \times h_{fg} \text{ at } 450 \text{ kPa}$$

$$2\,645{,}52 = 624 + x_2 \times 2\,120$$

$$x_2 = 0{,}954$$

$$V_2 = x_2 \times V_g$$

$$= 0{,}954 \times 0{,}4138 \text{ at } 450 \text{ kPa}$$

$$= 0{,}3946 \text{ m}^3/\text{kg}$$

$$A_2 = \frac{mV_2}{C_2}$$

$$= \frac{330{,}72 \times 0{,}3946}{60 \times 719}$$

$$= 3\,024{,}914 \text{ mm}^2$$

Example 1.6

Steam enters a converging-diverging nozzle at a pressure of 2,5 MPa and leaves at 350 kPa. The steam flow rate is 15 kg/s. The pressure at the throat is 0,54 times the inlet pressure, the temperature is 226,6 °C, the specific heat capacity of the superheated steam is 2,5 kJ/kg.K and the velocity is 529,15 m/s. The flow is frictionless up to the throat, but there is an 8% loss in the diverging part. The isentropic dryness fraction of the steam at exit is 0,95.

The index (n) for superheated steam is 1,3. Ignore the velocity at the entrance of the nozzle and calculate by using steam tables only:

1. Calculate the pressure of the superheated steam in kPa and the specific enthalpy in kJ/kg at the throat.
2. Calculate the specific enthalpy of the steam in kJ/kg at the entrance.
3. Calculate the specific volume in cubic metres per kg, the area in mm^2 and the diameter in millimetres at the throat.
4. Calculate the isentropic enthalpy in kJ/kg, the actual enthalpy in kJ/kg and the velocity in m/s at the exit of the nozzle.

Solutions

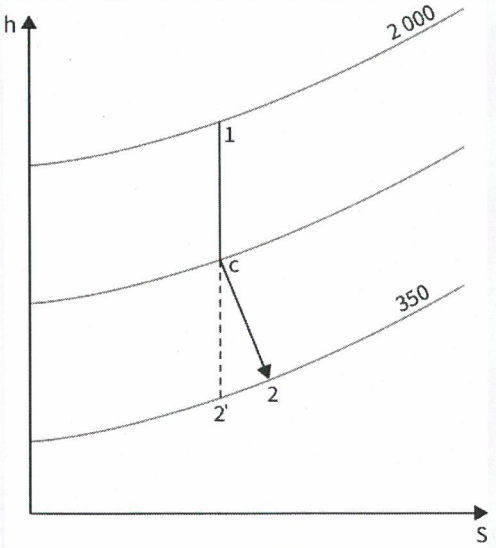


Figure 3.12

$P_1 = 2\ 500$ kPa
$P_2 = 350$ kPa
$m = 15$ kg/s
$P_c = 0{,}54\ P_1$
$t_c = 226{,}6°C$

$C_p = 25$ kJ/kg.K
$C_c = 529{,}15$ m/s
$\eta_d = 92\%$
$x_{2^1} = 0{,}95$
$n = 1{,}3$

1.

$$P_c = 0{,}54\ P_1$$
$$= 0{,}54 \times 2\ 500$$
$$= 1\ 350\ \text{kPa}$$

$$h_c = h_g + C_p(t_{s\mu} - t_s) \text{ at } 1\ 350\ \text{kPa}$$
$$= 2\ 787 + 2{,}5(2\ 26{,}6 - 1\ 934)$$
$$= 2\ 870\ \text{kJ/kg}$$

2.

$$C_c = \sqrt{2 \times 10^3(h_1 - h_c)}$$
$$529{,}15 = \sqrt{2 \times 10^3(h_1 - 2\ 780)}$$
$$h_1 = 3\ 010\ \text{kJ/kgs}$$

3.

$$V_c = \frac{n-1}{n}\left(\frac{h_1 - 1\ 941}{P_c}\right)$$
$$= \frac{0{,}3}{1{,}3}\left(\frac{2\ 780 - 1\ 941}{1\ 350}\right)$$
$$= 0{,}159\ \text{m}^3/\text{kg}$$

$$A_c = \frac{mV_c}{C_c}$$
$$= \frac{15 \times 0{,}159}{529{,}14}$$
$$= 4\ 501{,}656\ \text{mm}^2$$

$$d_c = \sqrt{\frac{4Ac}{\pi}}$$
$$= \sqrt{\frac{4 \times 4\ 501{,}656}{\pi}}$$
$$= 75{,}708\ \text{mm}$$

4.

$$h_{2'} = h_f + xh_{fg} \text{ at kPa}$$
$$= 584 + 0{,}95 \times 2\ 147$$

$$= 2\,623{,}65 \text{ kJ/kg}$$

$$\eta_d = \frac{h_c - h_2}{h_c - h_{2'}}$$

$$0{,}92 = \frac{2\,870 - h_2}{2\,870 - 2\,623{,}65}$$

$$h_2 = 2\,643{,}358 \text{ kJ/kg}$$

$$C_2 = \sqrt{(2 \times 10^3 (h_1 - h_2))}$$

$$= \sqrt{(2 \times 10^3 (3\,010 - 2\,643{,}358))}$$

$$= 856{,}32 \text{ m/s}$$

Example 1.7

Superheated steam enters a convergent-divergent nozzle at a pressure of 1,5 MPa and temperature of 230°C. The exit pressure is 700 kPa. The overall enthalpy drop is 90% of the isentropic enthalpy drop. The throat pressure is 1,15 MPa with a steam temperature of 189°C. The specific heat capacity of the steam at the throat pressure is 2,5 kJ/kgK. The isentropic enthalpy drop through the nozzle is 215 kJ/kg steam. The mass flow rate of the steam is 8 kg/s. The index (n) for superheated steam is 1,3. Ignore the steam velocity at the inlet.

Calculate the following:

1. steam velocity at the throat
2. throat diameter
3. isentropic and actual dryness fractions at the exit
4. exit diameter

Solutions

1. The steam tables have a pressure of 1,5 MPa, but no temperature of 230°C, and the specific heat capacity of the steam is not given. Therefore some interpolation is required.
The table below is an extract from the steam tables. You need to select the row where the pressure is given and then work on the two columns where the one value is below the given temperature and the other value is above the given value.

P (kPa)	200°C	250°C
1 500	2 796	2 925

$$h_1 = h_o + \Delta t_n \times \frac{\Delta h_o}{\Delta t_o}$$
$$= 2\,796 + (230 - 200) \times \frac{(2\,925 - 2\,796)}{(250 - 200)}$$
$$h_1 = 2\,873{,}4 \text{ kJ/kg}$$

$$h_c = h_g + C_p(t_{su} - t_s) \text{ @ } 1\,150 \text{ kPa}$$
$$= 2\,781 + 2{,}5(189 - 186{,}1)$$
$$h_c = 2\,788{,}25 \text{ kJ/kg}$$

$$C_c = \sqrt{2 \times 10^3 (h_1 - h_c)}$$
$$= \sqrt{2 \times 10^3 (2\,873{,}4 - 2\,788{,}25)}$$
$$C_c = 412{,}674 \text{ m/s}$$

2.

$$V_c = \frac{(n-1)}{n}\left(\frac{h_c - 1941}{P_c}\right)$$
$$= \frac{0{,}3}{1{,}3} \times \left(\frac{2\,788{,}25 - 1941}{1\,150}\right)$$
$$V_c = 0{,}17 \text{ m}^3/\text{kg}$$

$$A_c = \frac{\dot{m}V_c}{C_c} = \frac{8 \times 0{,}17}{412{,}674} = 3{,}296 \times 10^{-3}\ m^2$$

$$d_c = \sqrt{\frac{4A_c}{\pi}} = \sqrt{\frac{4 \times 3{,}296 \times 10^{-3}}{\pi}} = 64{,}78\ mm$$

3.

$$h_2 = h_1 - \Delta h = 2873{,}4 - 215 = 2\ 658{,}4\ kJ/kg$$

$$x_2 = \frac{h_2 - h_f}{h_{fg}}\ @700\ kPa$$

$$= \frac{2\ 658{,}4 - 697}{2\ 065}$$

$$x_2 = 0{,}95$$

$$\eta_n = \frac{h_1 - h_2}{h_1 - h_{2'}}$$

$$0{,}9 = \frac{2873{,}4 - 2658{,}4}{2873{,}4 - h_{2'}}$$

$$h_{2'} = 2\ 634{,}511\ kJ/kg$$

$$x_{2'} = \frac{h_{2'} - h_f}{h_{fg}}\ @700\ kPa$$

$$= \frac{2\ 634{,}511 - 697}{2\ 065}$$

$$x_{2'} = 0{,}938$$

Exercise 3.2

1. Steam enters a group of convergent-divergent nozzles at a pressure of 2 MPa and a temperature of 250 °C.
The velocity of the dry-saturated steam at the throat of the nozzle is 498 m/s and the diameter of the throat is 66,5 mm.

Expansion through the nozzle takes place to an exit pressure of 380 kPa. The flow is frictionless and adiabatic up to the throat. The efficiency from the throat to the exit is 90%.
At the exit, the isentropic dryness factor of the steam is 0,93.
Ignore the velocity of the steam at the inlet. Use only the steam tables to answer the questions.

1.1 Calculate the specific enthalpy of the steam at the throat and then use steam tables to determine the pressure, the temperature and the volume at the throat. 1.2 Calculate the area of the throat and the flow rate of the steam in kg/s.
1.3 Calculate the isentropic enthalpy and the actual enthalpy at the exit in kJ/kg. 1.4 Calculate the velocity in m/s, the actual dryness factor of the steam, the volume in cubic metres per kilogram and the area at the exit.

2. Steam expands through a convergent-divergent nozzle at a rate of 5 kg/s
to the exit, where the isentropic dryness factor is 0,94 and the diameter is 72,2 mm.
At the entrance, the superheated steam has a pressure of 1 500 kPa and a
temperature of 250 °C; velocity is negligible. At the throat, the superheated steam has a pressure of 820 kPa and a velocity of 500 m/s.
The specific heat capacity is 2,56 kJ/kg.K with an index (n) of 1,31.
The specific volume of dry-saturated steam at the exit pressure is 0,6684 m^3/kg. The isentropic dryness factor is 98,95% of the actual dryness factor.
Use only the steam tables to answer the questions.

2.1 Calculate the specific enthalpy of the superheated steam in kJ/kg and the temperature in °C at the throat.
2.2 Calculate the specific volume in m^3/kg, the area in mm^2 and the diameter in mm at the throat.
2.3 Calculate the actual dryness factor, the specific volume in m^3/kg, the area in mm^2, the velocity in m/s and the specific actual enthalpy in kJ/kg at the exit.
3. A convergent-divergent nozzle receives superheated steam at a pressure of 800 kPa and a temperature of 200 °C, and discharges it at a pressure of 200 kPa. The change of specific enthalpy of the steam is 190 kJ and the exit area of the nozzle is 6 450 mm^2. At the throat the steam is dry-saturated, with a velocity of 424,264 m/s. Ignore the velocity of the steam at the inlet.
Use only the steam tables to answer the questions.

3.1 Calculate the dryness fraction, the velocity in m/s and the mass of the steam in kg/s at the outlet of the nozzle.
3.2 Calculate the specific enthalpy, the pressure in kPa, the temperature in degrees Celsius, the area in mm^2 and the diameter in mm at the throat of the nozzle.
3.3 Calculate the specific heat capacity of the steam at the nozzle inlet.

Module summary

Gas nozzles

$$T_{2'} = T_c \times \left(\frac{P_2}{P_c}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = T_c \times \left(\frac{P_2}{P_c}\right)^{\frac{n-1}{n}}$$

$$T_{2'} = T_1 \times \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = T_1 \times \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

$$T_c = T_1 \times \left(\frac{P_c}{P_1}\right)^{\frac{n-1}{n}}$$

$\eta_c = \frac{T_1 - T_c}{T_1 - T_{c'}}$ (converging efficiency)

$\eta_d = \frac{T_c - T_2}{T_c - T_{2'}}$ (diverging efficiency)

$\eta_n = \frac{T_1 - T_2}{T_1 - T_{2'}}$ (nozzle efficiency)

$$C_c = \sqrt{2C_p(T_1 - T_c)}$$

$$C_2 = \sqrt{2C_p(T_1 - T_2)}$$

$$A = \frac{\dot{m}V}{C}$$

Steam nozzles

$\eta_c = \frac{h_1 - h_c}{h_1 - h_{c'}}$ (converging efficiency)

$\eta_d = \frac{h_c - h_2}{h_c - h_{2'}}$ (diverging efficiency)

$\eta_n = \frac{h_1 - h_2}{h_1 - h_{2'}}$ (nozzle efficiency)

$$C_c = \sqrt{2 \times 10^3(h_1 - h_c)}$$

$$C_2 = \sqrt{2 \times 10^3(h_1 - h_2)}$$

$$V_{su} = \frac{(n-1)}{n}\left(\frac{h_{su}-1941}{P_c}\right)$$

$$A = \frac{\dot{m}V}{C}$$

Summative assessment: Module 3

1. Steam expands through a convergent-divergent nozzle to the exit where the isentropic dryness factor is 0,94 and the diameter is 72,2 mm. At the entrance the steam has a pressure of 1,5 MPa and a temperature of 250°C. At the throat the steam has a pressure of 820 kPa, a specific heat capacity of 2,56 kJ/kgK, a velocity of 500 m/s and an index of expansion of 1,31. The exit pressure is 270 kPa and the isentropic dryness factor is 98,95% of the actual dryness factor.

Calculate the following:

1.1 temperature at the throat	[5]
1.2 diameter at the throat	[5]
1.3 specific volume at the exit	[4]
1.4 velocity at the exit	[4]
1.5 mass flow rate.	[2]

2. Air enters a convergent-divergent nozzle at a temperature of 450° and pressure of 2,8 MPa. The diameter is 51 mm at the throat. The exit pressure is 590,674 kPa. The flow is frictionless up to the throat. The diverging efficiency is 96%. The value of the gas constant R is 287 J/kgK and C_v is 718 J/kgK.

Calculate the following:

2.1 specific heat ratio	[2]
2.2 velocity at the throat	[5]
2.3 mass flow rate	[5]

2.4 the Mach number. [8]

3. Air flows through a convergent-divergent nozzle at a mass rate of 150 kg/min to and exit pressure of 237,26 kPa. The pressure at the throat is 475,454 kPa with an area of 1 534,6 mm^2. The actual temperature in the throat is 131,11°C which results in a velocity of 398,801 m/s. The nozzle efficiency is 92%. The specific heat ratio for air is 1,4 and C_p is 1,008 kJ/kgK.

Calculate the following:

3.1 temperature and pressure at inlet [4]
3.2 percentage heat loss in the converging section [6]
3.3 velocity at exit [4]
3.4 area at exit [4]
3.5 the Mach number. [2]

4. A convergent-divergent nozzle receives steam at a pressure of 800 kPa and 200°C and discharges at 200 kPa. The area at the exit is 6 450 mm^2. The overall change in specific enthalpy of the steam is 190 kJ/kg.. At the throat the steam is dry saturated with a velocity of 424,264 m/s.

Calculate the following:

4.1 dryness fraction at the exit [5]
4.2 mass flow rate [5]
4.3 pressure at the throat using steam tables [4]
4.4 diameter at the throat [4]
4.5 specific heat capacity at inlet. [2]

5. A convergent-divergent nozzle expands steam to an exit pressure of 400 kPa and a velocity of 700 m/s. The condition of the steam at exit is 2,719% wet. At a throat pressure of 820 kPa and temperature 180,2°C the specific heat capacity is 2,5 kJ/kg. The index for superheated steam is 1,3. The throat has a diameter of 55 mm. Flow is frictionless up to the throat. The diverging efficiency is 88%.

Calculate the following:

5.1 pressure at inlet using steam tables	[6]
5.2 velocity at the throat	[4]
5.3 mass flow rate	[6]
5.4 isentropic dryness of steam at exit.	[4]

Total: 100 marks

MODULE

4 Steam generation and nuclear power plants

By the end of this module, students should be able to:

- analyse given data to determine specific enthalpy at inlet and exit points of the steam generation plant units using the steam tables;
- perform all relevant calculations on the air preheater;
- perform calculations on efficiencies;
- perform calculations and/or draw up detailed energy balance table with respect to the:
  - heat supplied
  - heat transferred to the economiser
  - heat transferred to the evaporator
  - heat transferred to the superheater
  - heat lost to moisture content
  - heat lost to flue gases
  - radiation;
- apply the following formulae for:
  - heat lost to moisture content;
  - heat lost to flue gases, namely the mass of:
    - mass of flue gases
    - mass of dry gases;
- explain the principle of operation of turbine generator in a conventional island; and
- perform power and thermal efficiency calculations according to:
  - ideal Rankine cycle
  - actual Rankine cycle.

Introduction

The process of steam generation involves boiling water to produce steam, which undergoes a phase change from liquid to gas due to applied heat. This heat must be consistently supplied for the conversion to occur.

A basic steam generation setup includes a kettle type boiler that heats a specific amount of water. As heat is applied, the water temperature rises until it reaches the boiling (saturation) point, leading to the formation of steam bubbles. At this stage, all the incoming heat energy is utilized for vaporizing water rather than increasing temperature.

Continuous steam production requires a regulated water supply, typically facilitated by a pump to match the steam output. Steam heated beyond the saturation point is termed superheated steam, while cooled water below saturation is referred to as sub cooled water.

Understanding these principles is fundamental for designing and operating steam generation systems in mechanical engineering contexts.

4.1 Steam plant concepts

Central to understanding the operation of steam generation is the basic steam cycle.

Basic steam cycle

It is a process in which steam is generated in a boiler at a constant high pressure, expanded through a turbine to a low pressure to extract work, condensed into water in a condenser at a constant low

pressure and finally fed back to the boiler at high pressure by a centrifugal pump.

The general arrangement of the basic elements of a steam plant is illustrated in Figure 4.1.

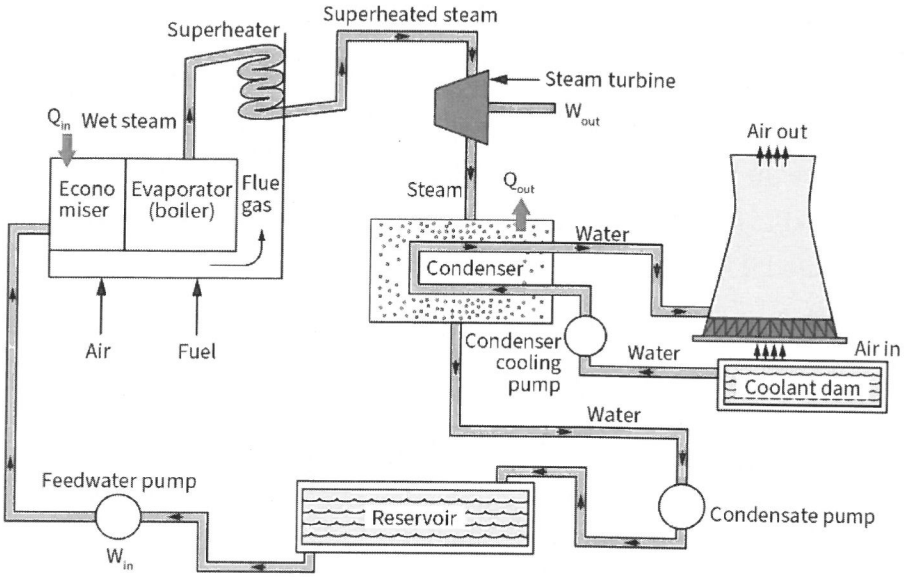


Figure 4.1: Basic elements of a steam plant

Phases of steam generation

Steam is a gaseous fluid that is formed when water evaporates. The gaseous particles that are formed are called vapour or steam. This distinguishes them from gas. Gases are vapours at a much higher temperature than that at which they were evaporated.

All gases or vapours have the property to increase their volume or to absorb heat. When water boils at sea level the pressure on the surface of the water is at standard atmospheric pressure of 101,325 kPa above absolute zero pressure. At this condition the temperature of the steam produced from boiling water is 100 °C.

If the pressure on the surface of the boiling water changes, the boiling point temperature of water also changes. The enthalpy of water is its total heat content in joules above 0°C and is measured in J/kgK. The three phases of steam generation are:

Sensible heat or liquid enthalpy (h_f)
It is the amount of heat required to change the temperature of water from 0°C for a unit mass and is measured in J/kg. It depends on the pressure on the surface of the water. Steam tables are used to look up the specific liquid enthalpy of water at a certain pressure or it can be calculated if the specific heat capacity at the given temperature is known.

$$h_f = c_w t_w$$

Where:
h_f is specific liquid enthalpy (J/kg)
c_w is specific heat capacity for water (J/kg°C)
t_w is temperature of water (°C)

Latent heat or enthalpy of evaporation (h_{fg})
It is the amount of heat required to change the phase from a saturated liquid to a saturated vapour at constant temperature for a unit mass and is measured in J/kg. It depends on the pressure on the surface of the water. Steam tables are used to look up the specific liquid enthalpy of water at different pressures.

Superheat or enthalpy of superheat (h_{su})
It is the amount of heat required to increase the temperature of the saturated steam to a higher value than its saturation temperature for a

unit mass and is measured in J/kg. Tables for superheated steam can be used to look up the specific superheat enthalpy at the specific pressure and superheated temperature. Normal steam tables can also be used to look up the liquid enthalpy and enthalpy of evaporation and can then be calculated if the specific heat capacity of the superheated steam is known.

$$h_g = h_f + h_{fg}$$

$$h_{su} = h_g + c_{su}(t_{su} - t_s)$$

Where:
h_g is specific enthalpy of saturated vapour (J/kg)
h_{su} is specific enthalpy of superheat (J/kg)
c_{su} is specific heat capacity for superheated vapour (J/kg°C)
t_{su} is superheated temperature (°C)
t_s is saturated temperature (°C)

Figure 4.2 shows the three phases of steam generation from liquid water to superheated steam in a temperature-specific enthalpy (t-h) diagram.

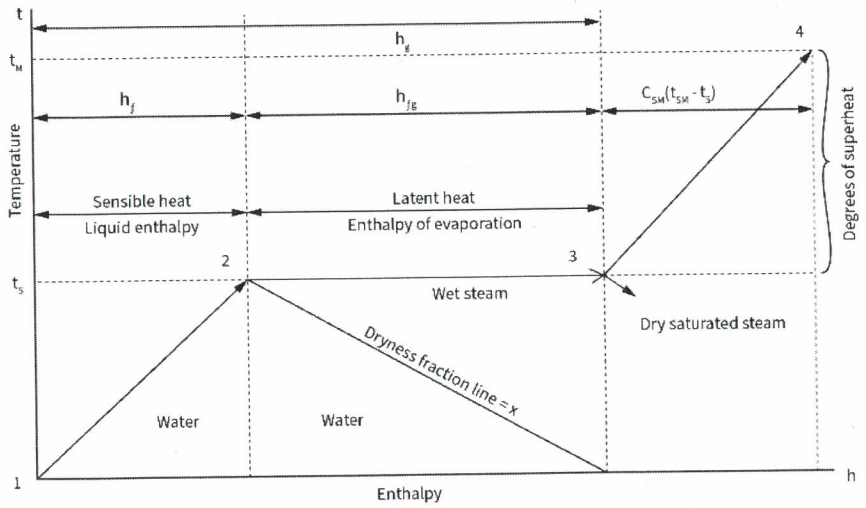


Figure 4.2: t-h diagram of steam generation

Process 1-2 (Sensible heat)

The position of point 1 in the diagram starts at 0°C where the specific liquid enthalpy of water is zero. The position of point 1 could be higher than 0°C which means that the specific liquid enthalpy at point 1 will then also need to be calculated, depending on its temperature.

Process 2-3 (Latent heat)

The position of point 3 in the diagram shows that the steam is dry saturated which means that all the water particles in the water has been converted into steam. The position of point 3 could be to the left which means it is a vapour that still contains some moisture.

The dryness fraction line represents the ratio of water to steam during the process of evaporation. While the water boils, more and more steam is formed and less water remains. If the dryness fraction of steam is 0,8, it means that 80% of the water has evaporated, and 20% water still needs to evaporate This condition is known as wet steam. If

the steam is still wet, the dryness fraction will be given in order to calculate the specific enthalpy of evaporation.

Note
Please note that the dryness fraction for saturated steam is 1.

$h_e = x \times h_{fg}$

Process 3-4 (Superheat)
The saturation temperature depends on the pressure on the water and the superheated temperature depends on the degrees of superheat required.

Main components
The main components that have to do with the steam generation process are shown as a line diagram in Figure 4.3. The line diagram shows the paths followed by the water, flue gas and air.

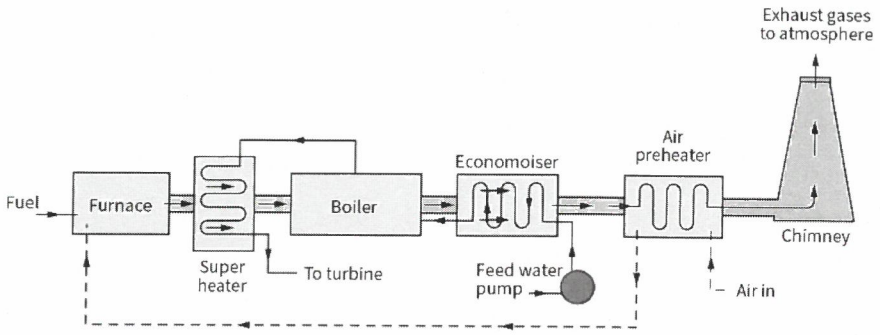


Figure 4.3: Line diagram showing fluid paths

Air preheater
It is basically a heat-exchanger installed in the exit flue gas duct of the boiler. The purpose of the air preheater is to recover the heat from the

boiler flue gas to increase the temperature of the air that is used for combustion in the furnace.

The air preheater is placed in the path of the flue gasses and is the last component before the flue gas enters the chimney. The thermal efficiency of the plant is increased by reducing the useful heat lost in the flue gas.

Economiser

It is also known as a feedwater heater. It is a device in which the waste heat of the flue gases is used for heating the feed water before it enters the boiler or evaporator. The economiser is placed in the path of flue gases just before the air preheater. The thermal efficiency of the plant is increased by reducing the useful heat lost in the flue gas.

Superheater

It is used to dry the wet steam and to increase the temperature of the steam to above its saturation temperature. It is paced in the path of the flue gases after the boiler in order to absorb heat from the flue gasses. The thermal efficiency of the plant is increased by reducing the useful heat lost in the flue gas.

Evaporator

It is also known as the boiler. The evaporator is an essential component of a steam power plant as it is responsible for converting the liquid water into steam that is used to drive the turbines. The evaporator typically consists of a series of tubes through which the water flows.

These tubes or coils are heated by the hot gases produced by the combustion of fuel in the furnace. As the water flows through the

tubes, it absorbs heat from the hot gases and begins to boil. This causes the water to evaporate and turn into steam.
The steam is then transferred to different types of mechanical machines which converts the thermal energy of the steam into useful mechanical work.

Heat absorbed by each component
In order to calculate the heat absorbed by the four main components in the steam plant it needs to be understood that the heat that is absorbed by each component is assumed to be the same as the heat that is rejected by the flue gas flowing over each component.
It is therefore assumed that the heat transfer is 100% effective in each component.

The heat absorbed by the water and steam is calculated by considering the change in enthalpy from the inlet to the outlet of each component. The heat rejected by the flue gas is calculated by considering the change in temperature from the inlet to the outlet over each component.

$Q_a = m_s \times \Delta h$ and $Q_r = m_g C_g \times \Delta t$

Due to the fact that there is saturated liquid, wet steam and superheated conditions in the steam production line, it is important to know the sequence where each component is. The same applies to the sequence of how the flue gas flows over each component through the steam plant.

A line diagram to show these paths is shown in Figure 4.4.

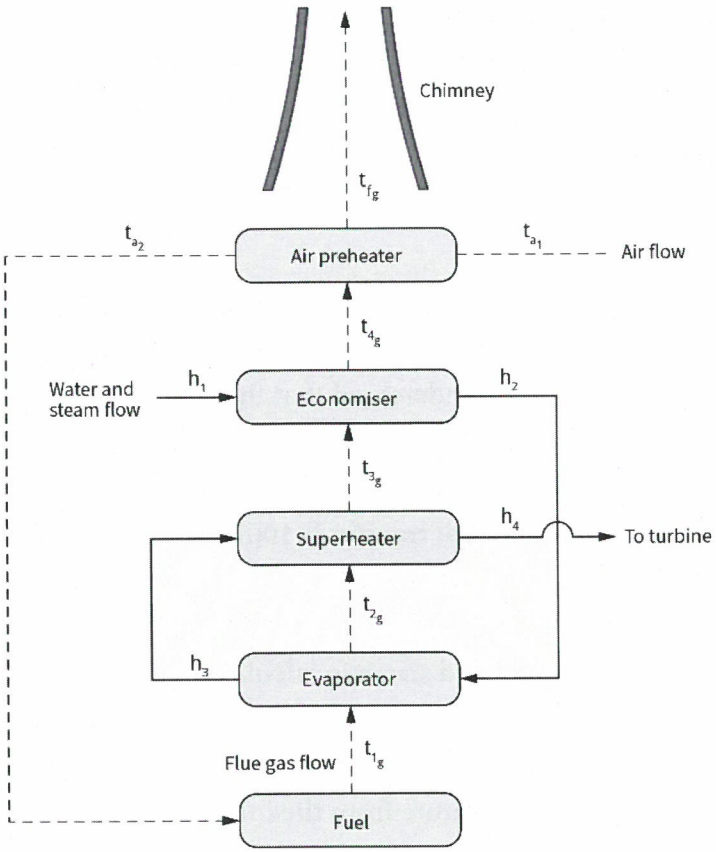


Figure 4.4: Steam plant flow diagram

Air preheater

$$Q_{ap} = m_a C_a (t_{a2} - t_{a1}) \text{ or } Q_{ap} = m_g C_g (t_{4g} - t_{fg})$$

Economiser

$$Q_{ec} = m_s (h_2 - h_1) \text{ or } Q_{ec} = m_g C_g (t_{3g} - t_{4g})$$

Superheater

$$Q_{su} = m_s (h_4 - h_3) \text{ or } Q_{su} = m_g C_g (t_{2g} - t_{3g})$$

Evaporator

$$Q_{ev} = m_s(h_3 - h_2) \text{ or } Q_{ev} = m_g C_g(t_{1g} - t_{2g})$$

Heat energy supplied by fuel

$$Q_f = m_f \times CV$$

Thermal efficiency

$$\eta_t = \frac{Q_s}{Q_f} = \frac{m_s(h_4 - h_1)}{m_f \times CV}$$

The masses in the calculations could be given in different units, which will then influence the unit of the heat flow. If the mass of steam is given in kg per kg fuel, the unit for heat flow will be J/kg_f. If the mass of steam is given as a rate in kg per second, the unit for heat flow will be J/s. The mass of air is usually given as an air/fuel ratio like 20:1.

This means that for every 1 kg of fuel that is burned, 20 kg of air is supplied. The ratio of the flue gas/fuel will then be 21:1. This means that for every 1 kg of fuel that is burned, 21 kg of flue gas will be produced and will eventually flow through the chimney or flue stack.

The formula below shows the mass relations between the flue gas, the air and the fuel.

$$m_g = m_a + m_f$$

Example 4.1

A steam plant produces steam at 3 MPa and 400°C at a mass rate of 10 kg/s. The dryness fraction of the steam at the exit of the evaporator is 0,8. The feedwater enters the economiser at 13°C and leaves at 59°C. The air for combustion is supplied at a rate of 30 kg/s. The air

enters the preheater at 15°C and leaves at 35°C. The specific heat capacity for air is 1 kJ/kg°C. Fuel with a calorific value of 33 MJ/kg is used at a rate of 72 kg/min.

Calculate the following:
1. heat absorbed by the air preheater
2. heat absorbed by the economiser
3.) heat absorbed by the superheater
4. heat absorbed by the evaporator
5. thermal efficiency

Solutions

1.

$$Q_a = \dot{m}_a C_a (t_{a2} - t_{a1}) = 30 \times 1(35 - 15) = 600 \text{ kJ/s}$$

2.

$$h_1 = h_f@13°C = 55 \text{ kJ/kg}$$

$$h_2 = h_f@59°C = 247 \text{ kJ/kg}$$

$$Q_{ec} = \dot{m}_s(h_2 - h_1) = 10(247 - 55) = 1\,920 \text{ kJ/s}$$

3.

From steam tables at 3 000 kPa

$$h_3 = h_f + xh_{fg} = 1\,008 + 0{,}8 \times 1\,794 = 2\,443{,}2 \text{ kJ/kg}$$

From superheated tables at 3000 kPa and 400°C

$$h_4 = h_{su} = 3\,231 \text{ kJ/kg}$$

$$Q_{su} = \dot{m}_s(h_4 - h_3) = 10(3\,231 - 2\,443{,}2) = 7\,878 \text{ kJ/s}$$

4.

$$Q_{ev} = \dot{m}_s(h_3 - h_2) = 10(2\,443{,}2 - 247) = 21\,962 \text{ kJ/s}$$

5.

$$Q_f = m_f \times CV = 1{,}2 \times 33 \times 10^6 = 39\,600 \text{ kJ/s}$$

$$\eta_t = \frac{\dot{m}_s(h_4 - h_1)}{\dot{m}_f \times CV} = \frac{10(3\,231 - 55)}{39\,600} = 0{,}8 = 80\%$$

Heat balance

The heat balance of a steam plant refers to the systematic accounting of all heat inputs and outputs in a steam generation system. This concept is critical for ensuring the efficiency and safety of steam plants, as it helps to verify that energy is conserved within the system according to the first law of thermodynamics.

A typical heat balance table is shown below:

	Energy	**Percentage**
Energy supplied	a) Fuel	100%
Energy absorbed	b) Superheater	
	c) Evaporator	
	d) Economiser	
	e) Air preheater	
	f) Flue gas (dry)	
	g) Flue gas (wet)	
	h) Radiation	
	Total	100%

Formulae for heat balance

a) $Q_f = m_f \times CV$

b) $Q_{su} = m_s(h_4 - h_3)$

c) $Q_{ev} = m_s(h_3 - h_2)$

d) $Q_{ec} = m_s(h_2 - h_1)$

e) $Q_a = m_a C_a(t_{a2} - t_{a1})$

f) $Q_{dg} = m_{dg}C_g(t_g - t_{a1})$
g) $Q_m = m_m(h_{su} - h_f)$
h) $Q_r = a - (b + c + d + e + f + g)$

Where:
h_1 is specific enthalpy at inlet of economiser
h_2 is specific enthalpy at outlet of economiser
h_3 is specific enthalpy at outlet of evaporator
h_4 is specific enthalpy at outlet of superheater
h_{su} is specific enthalpy of superheated moisture at chimney base
h_f is specific enthalpy of water at atmospheric temperature
t_{a1} is atmospheric temperature
t_{a2} is air temperature at exit of air preheater
t_g is flue gas temperature at chimney base
C_a is specific heat capacity of air
C_g is specific heat capacity of flue gas
m_f is mass of fuel
m_s is mass of steam
m_a is mass of air
m_g is mass of flue gas
m_{dg} is mass of dry flue gas
m_m is mass of moisture in flue gas

The masses in the calculations could be given in different units, which will then influence the unit of the heat flow. If the mass of steam is given in kg per kg fuel, the unit for heat flow will be J/kg_f. If the mass of steam is given as a rate in kg per second, the unit for heat flow will be J/s. The mass of air is usually given as an air/fuel ratio, for instance 20:1.

This means that for every 1 kg of fuel that is burned, 20 kg of air is supplied. The ratio of the flue gas/fuel will then be 21:1. This means that for every 1 kg of fuel that is burned, 21 kg of flue gas will be produced and will eventually flow through the chimney or flue stack.

The formula below shows the mass relations between the fuel, air and flue gas.

$$m_g = m_a + m_f$$

$$m_m = 9 \times \%H_2 \text{ (according to combustion)}$$

$$m_{dg} = m_a + m_f - m_m$$

Example 4.2

A steam plant produces steam at 2 MPa and 350°C at a mass rate of 16 kg/s. The feedwater enters the economiser at 24,1°. The air enters the preheater at 25°C and leaves at 150°C. The air/fuel ratio is 20:1. The temperature at the chimney base is 101°C. The specific heat capacity for air is 1 kJ/kgK. The specific heat capacity for the flue gas is 1,05 kJ/kgK. Fuel with a calorific value of 33 MJ/kg is used at a rate of 108 kg/min.

Calculate the following:

1. heat supplied by the fuel per second
2. heat absorbed by the air preheater per second
3. heat absorbed by the boiler per second
4. heat absorbed by the flue gas per second
5. heat lost to radiation per second
6. percentage heat lost due to radiation by setting up a heat balance table

Solutions

1.

$Q_f = \dot{m}_f \times CV = 1{,}8 \times 33 \times 10^6 = 59{,}4\ MJ/s$

2.

$\dot{m}_a = \dot{m}_f \times a/f\ ratio = 1{,}8 \times 20 = 36\ kg/s$

$Q_{ap} = \dot{m}_a C_a (t_{a2} - t_{a1}) = 36 \times 1(150 - 25) = 4{,}5\ MJ/s$

3.

$h_1 = h_f\ @\ 24{,}1°C = 101\ kJ/kg$

From superheated tables at 2 000 kPa and 350°C

$h_4 = h_{su} = 3\ 138\ kJ/kg$

$Q_s = \dot{m}_s (h_4 - h_1) = 16(3\ 138 - 101) = 48{,}592\ MJ/s$

4.

$\dot{m}_g = \dot{m}_f + \dot{m}_a = 1{,}8 + 36 = 37{,}8\ kg/s$

$Q_g = \dot{m}_g C_g (t_g - t_{a1}) = 37{,}8 \times 1{,}05(101 - 25) = 3{,}016\ MJ/s$

5.

$Q_r = Q_f - Q_{abs}$

$= 59{,}4 - (48{,}592 + 4{,}5 + 3{,}016)$

$Q_r = 3{,}292\ MJ/s$

6.

	Energy (MJ/s)	**Percentage**
Energy supplied	a) $Q_f = 59{,}4\ MJ/s$	100%
Energy absorbed	b) $Q_s = 48{,}592\ MJ/s$	81,8%
	c) $Q_g = 3{,}016\ MJ/s$	5,08%
	d) $Q_a = 4{,}5\ MJ/s$	7,58%

	e) $Q_r = 3{,}292$ MJ/s	5,54%
	Total	100%

Example 4.3

A coal fired steam plant consists of an economiser, evaporator and superheater working at a pressure of 3 MPa. The steam is produced at 20 kg/s at a temperature of 350°C. The fuel with a calorific value of 34 MJ/kg is used at a rate of 120 kg/min. The feedwater enters the economiser at 31°C. The heat absorbed by the economiser is 4,76 MJ/s. The heat absorbed by the evaporator is 45,56 MJ/s. The pressure and temperature at the chimney base is 100 kPa and 250°C. The boiler room temperature is 24,1°C. The a/f ratio is 18:1. The specific heat capacity for the flue gas is 1,05 kJ/kgK. The fuel contains 4% hydrogen in the formation of moisture in the flue gas.

Calculate the following:

1. thermal efficiency of the plant
2. specific enthalpy at the entrance of the evaporator
3. specific enthalpy at the exit of the evaporator
4. dryness fraction of the steam at exit of the evaporator
5. heat absorbed in superheater per kg_f
6. heat lost to moisture in flue gas per kg_f
7. heat lost to dry flue gas per kg_f
8. percentage heat lost due to radiation by setting up a heat balance table in MJ/kg_f

Solutions

1.

$h_1 = h_f$ @ 31°C = 130 kJ/kg

From superheated tables at 3 000 kPa and 350°C

$h_4 = h_{su} = 3\ 117\ \text{kJ/kg}$

$$Q_s = \dot{m}_s(h_4 - h_1) = 20(3\ 117 - 130) = 59{,}74\ \text{MJ/s}$$

$$Q_f = \dot{m}_f \times CV = 2 \times 36 \times 10^6 = 72\ \text{MJ/s}$$

$$\eta_t = \frac{Q_s}{Q_f} = \frac{59{,}74}{72} = 0{,}83 = 82{,}972\%$$

2.

$$h_2 = \frac{Q_{ec}}{\dot{m}_s} + h_1 = \frac{4{,}76 \times 10^3}{20} + 130 = 368\ \text{kJ/kg}$$

3.

$$h_3 = \frac{Q_{ev}}{\dot{m}_s} + h_2 = \frac{45{,}56 \times 10^3}{20} + 368 = 2\ 646\ \text{kJ/kg}$$

4.

From steam tables at 3 000 kPa

$$x = \frac{h_3 - h_f}{h_{fg}} = \frac{2\ 646 - 1\ 008}{1\ 794} = 0{,}913$$

5.

$$m_s = \frac{\dot{m}_s}{\dot{m}_f} = \frac{20}{2} = 10\ \text{kg/kg}_f$$

$$Q_{su} = m_s(h_4 - h_3) = 10(3\ 117 - 2\ 646) = 4{,}71\ \text{MJ/kg}_f$$

6.

From sup tables at 100 kPa and 250°C (chimney base)

$h_{su} = 2\ 975\ \text{kJ/kg}$

From steam tables at 24,1°C (boiler room)

$h_f = 101\ kJ/kg$

$m_m = 9 \times \%H_2 = 9 \times 0{,}04 = 0{,}36\ kg/kg_f$

$Q_m = m_m(h_{su} - h_f) = 0{,}36(2\,975 - 101) = 1{,}035\ MJ/kg_f$

7.

$m_{dg} = m_a + m_f - m_m = 18 + 1 - 0{,}36 = 18{,}64\ kg/kg_f$

$Q_{dg} = \dot{m}_{dg} C_g (t_g - t_{a1})$
$= 18{,}64 \times 1{,}05(250 - 24{,}1)$
$Q_{dg} = 4{,}421\ MJ/kg_f$

8.

$$Q_{ec} = \frac{Q_{ec}}{\dot{m}_f} = \frac{4{,}76}{2} = 2{,}38\ MJ/kg_f$$

$$Q_{ev} = \frac{Q_{ev}}{\dot{m}_f} = \frac{45{,}56}{2} = 22{,}78\ MJ/kg_f$$

$Q_r = Q_f - Q_{abs}$
$= 36 - (22{,}78 + 4{,}71 + 2{,}38 + 1{,}035 + 4{,}421)$
$Q_r = 0{,}674\ MJ/s$

	Energy (MJ/kg_f)	**Percentage**
Energy supplied	a) $Q_f = 36\ MJ/kg_f$	100%
Energy absorbed	b) $Q_{ev} = 22{,}78\ MJ/kg_f$	63,28%
	c) $Q_{su} = 4{,}71\ MJ/kg_f$	13,08%
	d) $Q_{ec} = 2{,}38\ MJ/kg_f$	6,61%
	e) $Q_m = 1{,}035\ MJ/kg_f$	2,88%
	f) $Q_{dg} = 4{,}421\ MJ/kg_f$	12,28%
	g) $Q_r = 0{,}674\ MJ/kg_f$	1,87%
	Total	100%

Exercise 4.1

1. The following information was obtained from a test on a steam plant:

- Rate of evaporation = 7 350 kh/h
- Operating pressure = 3 MPa
- Temperature of steam = 250°C
- Temperature of feed water = 37,7°C
- Temperature of water at economiser outlet = 99,6°C
- Mass of fuel used = 750 kg/h
- Temperature of flue gas at entry to economizer = 353,33°C
- Mass of air per kg of fuel burnt = 18 kg
- Specific heat capacity of flue gas = 1,045 kJ/kgK
- Thermal efficiency of plant = 80%
- Dryness factor of steam at entry to superheater = 0,975

Calculate the following:

1.1 heat absorbed by economiser per kg fuel

1.2 heat absorbed by evaporator per kg fuel

1.3 heat absorbed by superheater per kg fuel

1.4 calorific value of fuel

1.5 percentage heat lost due to radiation by setting up a heat balance table in MJ/kg_f

2. A coal fired steam plant consists of an economiser, evaporator and superheater working at a pressure of 2,5 MPa. The steam is produced at 19,5 kg/s at a temperature of 300°C. The fuel with a calorific value of 32,5 MJ/kg is used at a rate of 7 800 kg/h. The feedwater enters the economiser at 32,9°C. The heat absorbed by the economiser is 2,349 MJ/kg_f. The heat absorbed by the evaporator is 286,98 MJ/kg_f. The pressure and temperature at the chimney base is 100 kPa and 250°C. The boiler room temperature is 25°C. The a/f ratio is 18:1. The

specific heat capacity for the flue gas is 1,045 kJ/kgK. The specific heat capacity of water is 4,2 kJ/kgK. The fuel contains 4% hydrogen in the formation of moisture in the flue gas.

Calculate the following:
2.1 thermal efficiency of the plant
2.2 specific enthalpy at the entrance of the evaporator
2.3 specific enthalpy at the exit of the evaporator
2.4 dryness fraction of the steam at exit of the evaporator
2.5 heat absorbed in superheater per kg fuel
2.6 heat lost to moisture in flue gas per kg fuel
2.7 heat lost to dry flue gas per kg fuel
2.8 percentage heat lost due to radiation by setting up a heat balance table in MJ/kg_f

3. A steam plant with a thermal efficiency of 75% is fired with coal having a calorific value of 36 MJ/kg. Steam is generated at a rate of 9 kg per kg fuel burnt. The feed water enters the economiser at 32,9°C and leaves at 109,3°C. The dryness factor of the steam at the entrance to the superheater is 0,96. The flue gas at the chimney base has a temperature of 250°C and its specific heat capacity is 1,045 kJ/kgK. The pressure and temperature in the boiler room is 100 kPa and 22,6°C. The air/fuel ratio is 18:1. The heat carried away by the moisture in the flue gas is estimated as 1 152,032 kJ/kg of fuel burnt. The specific heat capacity of water is 4,2 kJ/kgK.

Calculate the following:
3.1 specific enthalpy of the superheated steam
3.2 pressure and temperature of the superheated steam
3.3 heat absorbed by the economiser per kg fuel
3.4 heat absorbed by the evaporator per kg fuel

3.5 heat absorbed by the superheater per kg fuel
3.6 the mass of the moisture in the flue gas
3.7 the heat lost in the dry flue gas per kg fuel
3.8 percentage heat lost due to radiation by setting up a heat balance table in MJ/kg_f

4.2 Nuclear plants

A nuclear power plant works to a large extent like a conventional thermal power plant. The only difference is that the heat, which converts water into steam, is produced by nuclear fission and not by burning coal, natural gas or liquid fuel.

The basic layout of a nuclear power plant is shown in Figure 4.5.

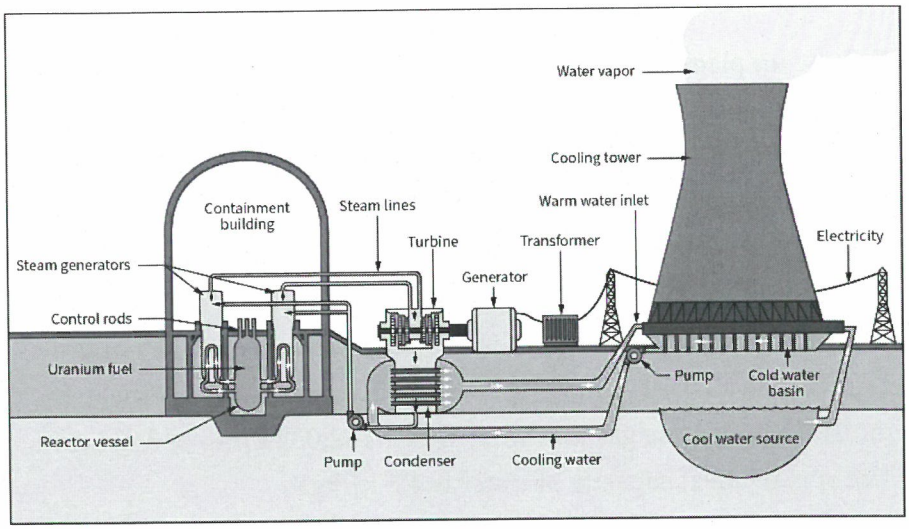


Figure 4.5: Layout of nuclear steam plant

The turbine generator in a conventional island operates by converting thermal energy into electrical energy through a series of well-defined processes. This system is integral to both nuclear and fossil-fuel power plants, where it performs a similar function.

4.2.1 Principles of turbine generator operation

The operation of the turbine generator follows the Rankine cycle, a thermodynamic cycle that describes how heat is converted into work. In a typical setup:

- **Steam generation**: Water is heated in the steam generator, converting it into high-pressure steam.
- **Turbine expansion**: The high-pressure steam expands through the turbine, causing it to rotate and perform mechanical work.
- **Energy conversion**: The mechanical energy from the turbine is converted into electrical energy by the generator.
- **Condensation and recirculation**: The steam is then condensed back into water in the condenser, completing the cycle and allowing the water to be reused in the steam generator.

This cycle is vital for maintaining a continuous and efficient power generation process in conventional islands, whether in nuclear or fossil-fuel power plants.

Steam generation (ideal Rankine cycle)

The Rankine cycle is the process widely used by power plants such as coal-fired power plants or nuclear reactors. A fuel is used to produce heat within a boiler, converting water into steam which then expands through a turbine producing useful work.

The line diagram of the Rankine process is shown in Figure 4.6 (a) and the ideal TS-diagram is shown in Figure 4.6 (b).

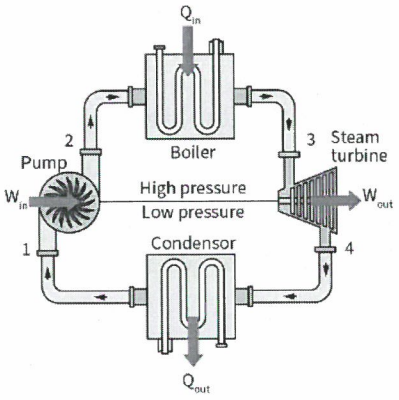


Figure 4.6(a) – Rankine cycle

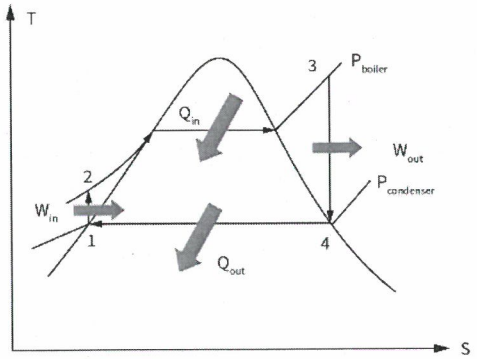


Figure 4.6(b) – TS-diagram

Process 1-2: The work input is provided by a centrifugal pump, water is isentropically compressed from a low pressure to a high pressure and forced into the boiler.

Process 2-3: Heat is added at constant pressure to heat the water and turn it into a wet vapour and finally into superheated vapour.

Process 3-4: The superheated vapour is allowed to expand isentropically from a high pressure to a low pressure through the turbine vanes which produce work out.

Process 4-1: Heat is removed from the vapour at constant pressure and the vapour is converted into a saturated liquid.

A system that uses the Rankine cycle has four main parts:

Pump

The function of the pump is to isentropically pump water from a low pressure to a high pressure into the boiler.

Boiler

The function of the boiler is to generate steam at constant pressure. The steam is used to drive machinery or turbines that are used to do work.

Turbine

The function of the turbine is to isentropically expand the high pressure superheated steam to a lower condenser pressure over its vanes to produce work in order to drive a generator that generates electrical power.

Condenser

The function of the condenser is to remove the heat from the incoming wet steam at constant pressure in order to condensate the steam to a saturated liquid so that it can be pumped back to the boiler.

4.2.2 Calculations

Process 1-2: Specific work provided by the pump

$$W_p = h_2 - h_1$$

OR

$$W_p = \bar{v}_1 \Delta p$$

Where

W_p is specific work done by pump (J/kg)

h_2 is specific enthalpy of water at exit of pump (J/kg)

h_1 is specific enthalpy of water at inlet of pump (J/kg)

$\bar{v}_1$ is specific volume of water at inlet (m^3/kg)

$$\bar{v}_1 = \frac{1}{\rho} \text{ (inverse of density)}$$

Δp is pressure difference between pump inlet and outlet (Pa)

Process 2-3: Specific heat energy supplied to water by boiler

$$q_i = h_3 - h_2$$

Where
q_i is specific input heat energy supplied by boiler (J/kg)
h_3 is specific enthalpy of steam at exit of boiler (J/kg)
h_2 is specific enthalpy of water at inlet of boiler (J/kg)

Process 3-4: Specific work done by turbine

$$W_t = h_3 - h_4$$

Where
W_t is specific work done by turbine (J/kg)
h_4 is specific enthalpy of steam at exit of turbine (J/kg)
h_3 is specific enthalpy of steam at inlet of turbine (J/kg)

Process 4-1: Heat energy rejected from water by condenser

$$q_o = h_4 - h_1$$

Where
q_o is specific output heat energy rejected by condenser (J/kg)
h_4 is specific enthalpy of steam at inlet of condenser (J/kg)
h_1 is specific enthalpy of water at outlet of condenser (J/kg)

Thermal efficiency

$$\eta_t = \frac{W_o}{q_i} = \frac{W_t - W_p}{q_i}$$

Where
W_o is the nett work done or work out (J/kg)

Example 4.4

A steam plant working on the ideal Rankine cycle has a boiler pressure of 3 MPa and a condenser pressure is 75 kPa. Both pressures are absolute values. The feed water enters the pump as a saturated liquid with a density of 975 kg/m^3. The boiler produces steam at a temperature of 350°C.

Calculate the following:

1. specific enthalpy at all the stages
2. specific work done by the pump
3. specific heat energy transfer in the boiler
4. specific work done by the turbine
5. specific heat energy transfer in the condenser
6. specific nett work done
7. thermal efficiency of the plant

Solutions

1.

$h_1 = h_f = 385\ \text{kJ/kg}\ @75\ \text{kPa}$

$$\Delta h = \bar{v}_1 \Delta p = \frac{1}{975} \times (2\ 925) = 3\ \text{kJ/kg}$$

$h_2 = h_1 + \Delta h = 385 + 3 = 388\ \text{kJ/kg}$

$h_3 = h_{su} = 3\ 117\ \text{kJ/kg}\ @3\ 000\ \text{kPa}\ \&\ 350°\text{C}$

$S_4 = S_1\ (\text{isentropic process})$

$S_4 = 6{,}774\ \text{kJ/kg}\ @\ 3\ 000\ \text{kPa}\ \&\ 350°\text{C}$

$S_f = 1{,}213\ \text{kJ/kg}\ @\ 75\ \text{kPa}$

$S_g = 7{,}457$ kJ/kg @ 75 kPa

$S_4 < S_g$ (this means that it is wet steam at condition 4)

$$x_4 = \frac{S_4 - S_f}{S_g - S_f} = \frac{6{,}774 - 1{,}213}{7{,}457 - 1{,}213} = 0{,}891$$

$h_4 = h_f + x_4 h_{fg}$@75 kPa
$= 385 + 0{,}891 \times 2\ 279$
$h_4 = 2\ 414{,}712$ kJ/kg

2.
$W_p = h_2 - h_1 = 388 - 385 = 3$ kJ/kg

3.
$q_i = h_3 - h_2 = 3\ 177 - 388 = 2\ 789$ kJ/kg

4.
$W_t = h_3 - h_4 = 3\ 177 - 2\ 414{,}712 = 762{,}288$ kJ/kg

5.
$q_o = h_4 - h_1 = 2\ 414{,}712 - 385 = 2\ 029{,}712$ kJ/kg

6.
$W_o = W_t - W_p = 762{,}288 - 3 = 759{,}288$ kJ/kg

7.
$$\eta_t = \frac{W_o}{q_i} = \frac{759{,}288}{2\ 789} = 0{,}272 = 27{,}224\%$$

Changing specific work to work rate (power) (J/s or W)

$$P = \dot{m} \times W$$

Changing specific heat energy to heat energy rate (J/s or W)

$Q = \dot{m} \times q$

Where

$\dot{m}$ is mass flow rate (kg/s)

Example 4.5

A steam plant working on the ideal Rankine cycle has a boiler pressure of 2 MPa and a condenser pressure is 10 kPa. Both pressures are absolute values. The feed water enters the pump as a saturated liquid with a density of 995 kg/m^3 and a mass flow rate of 25 kg/s The boiler produces steam at a temperature of 400°C.

Calculate the following:

1. specific enthalpy at all the stages
2. power of the pump
3. energy transfer rate in the boiler
4. power of the turbine
5. heat energy transfer rate in the condenser
6. nett power
7. thermal efficiency of the plant

Solutions

1.

$h_1 = h_f = 192 \text{ kJ/kg @10 kPa}$

$$\Delta h = \bar{v}_1 \Delta p = \frac{1}{995} \times (1\,990) = 2 \text{ kJ/kg}$$

$h_2 = h_1 + \Delta h = 192 + 2 = 194 \text{ kJ/kg}$

$h_3 = h_{su} = 3\,248 \text{ kJ/kg @2 000 kPa \& 400°C}$

$S_4 = S_1$ (isentropic process)

$S_4 = 7{,}126$ kJ/kg @ 2 000 kPa & 400°C

$S_f = 0{,}649$ kJ/kg @ 10 kPa

$S_g = 8{,}151$ kJ/kg @ 75 kPa

$S_4 < S_g$ (this means that it is wet steam at condition 4)

$$x_4 = \frac{S_4 - S_f}{S_g - S_f} = \frac{7{,}126 - 0{,}649}{8{,}151 - 0{,}649} = 0{,}863$$

$h_4 = h_f + x_4 h_{fg}$@10 kPa
$= 192 + 0{,}863 \times 2\,393$
$h_4 = 2\,258{,}044$ kJ/kg

2.
$P_p = \dot{m}(h_2 - h_1) = 25(194 - 192) = 50$ kW

3.
$Q_i = \dot{m}(h_3 - h_2) = 25(3\,248 - 194) = 76{,}35$ MJ/s

4.
$P_t = \dot{m}(h_3 - h_4) = 25(3\,248 - 2\,258{,}044) = 24{,}749$ MW

5.
$Q_o = \dot{m}(h_4 - h_1) = 25(2\,258{,}044 - 192) = 51{,}651$ MJ/s

6.
$P_o = P_t - P_p = 24{,}749 - 0{,}05 = 24{,}699$ MW

7.

$$\eta_t = \frac{P_o}{Q_i} = \frac{24{,}699}{76{,}35} = 0{,}323 = 32{,}35\%$$

In a real power-plant cycle (the name "Rankine" cycle is used only for the ideal cycle), the compression by the pump and the expansion in the turbine are not isentropic. In other words, these processes are non-reversible, and entropy is increased during the two processes. This somewhat increases the power required by the pump (due to isentropic efficiency) and decreases the power generated by the turbine (due to isentropic efficiency).

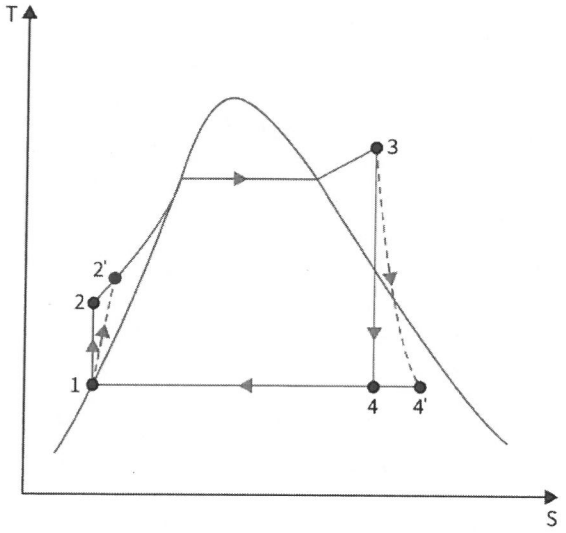


Figure 4.7: Practical Rankine cycle

Process 1-2': Actual work of pump

$W_{pa} = h_{2'} - h_1$

Where

W_{pa} is actual work provided by pump (J/kg)

$h_{2'}$ is actual specific enthalpy of water at exit of pump (J/kg)

Process 2'-3: Actual heat energy supplied by boiler

$q_{ia} = h_3 - h_{2'}$

Where

q_{ia} is actual heat energy supplied by boiler (J/kg)

$h_{4'}$ is actual specific enthalpy of steam at exit of turbine (J/kg)

Process 3-4': Actual work of turbine

$W_{ta} = h_3 - h_{4'}$

Where

W_{ta} is actual work developed by turbine (J/kg)

$h_{4'}$ is actual specific enthalpy of steam at exit of turbine (J/kg)

Process 4'-1: Actual heat energy rejected by condenser

$q_{oa} = h_{4'} - h_1$

Where

q_{oa} is actual heat energy rejected by condenser (J/kg)

$h_{4'}$ is actual specific enthalpy of steam at inlet of condenser (J/kg)

Actual thermal efficiency

$$\eta_{ta} = \frac{W_{oa}}{q_{ia}} = \frac{W_{ta} - W_{pa}}{q_{ia}}$$

Isentropic efficiency of pump

$$\eta_{ip} = \frac{h_2 - h_1}{h_{2'} - h_1} = \frac{P_{ps}}{P_{pa}}$$

Where

P_{ps} is pump power with isentropic values

P_{pa} is pump power with actual values

Isentropic efficiency of turbine

$$\eta_{it} = \frac{h_3 - h_{4'}}{h_3 - h_4} = \frac{P_{ta}}{P_{ts}}$$

Where:

P_{ts} is turbine power with isentropic values

P_{ta} is turbine power with actual values

Example 4.6

A steam plant working on the practical Rankine cycle has a boiler pressure of 8 MPa and a condenser pressure is 16 kPa. Both pressures are absolute values. The feed water enters the pump as a saturated liquid with a density of 998 kg/m^3 and a mass flow rate of 15 kg/s. The boiler produces steam at a temperature of 500°C. The pump has an isentropic efficiency of 80% and the turbine has an isentropic efficiency of 90%.

Calculate the following:

1. specific enthalpy at all the stages
2. power of the pump
3. heat energy transfer rate in the boiler
4. power of the turbine
5. heat energy transfer rate in the condenser
6. nett power
7. thermal efficiency of the plant

Solutions

1.

$h_1 = h_f = 232$ kJ/kg @16 kPa

$$\Delta h = \bar{v}_1 \Delta p = \frac{1}{998} \times (7\,984) = 8 \text{ kJ/kg}$$

$$h_2 = h_1 + \Delta h = 232 + 8 = 240 \text{ kJ/kg}$$

$$h_{2'} = h_1 + \frac{h_2 - h_1}{\eta_{ip}} = 232 + \frac{8}{0{,}8} = 242 \text{ kJ/kg}$$

$h_3 = h_{su} = 3\,398$ kJ/kg @8 000 kPa & 500°C

$S_4 = S_1$ (isentropic process)

$S_4 = 6{,}723$ kJ/kg @ 3 000 kPa & 500°C

$S_f = 0{,}772$ kJ/kg @ 10 kPa

$S_g = 7{,}987$ kJ/kg @ 75 kPa

$S_4 < S_g$ (this means that it is wet steam at condition 4)

$$x_4 = \frac{S_4 - S_f}{S_g - S_f} = \frac{6{,}723 - 0{,}772}{7{,}987 - 0{,}772} = 0{,}825$$

$h_4 = h_f + x_4 h_{fg}$@10 kPa

$= 232 + 0{,}825 \times 2\,370$

$h_4 = 2\,186{,}798$ kJ/kg

$$h_{4'} = h_3 - \eta_{ip}(h_3 - h_4)$$
$$= 3\,398 - 0{,}9(3\,398 - 2\,186{,}798)$$
$$h_{4'} = 2\,307{,}918\ \text{kJ/kg}$$

2.
$$P_{pa} = \dot{m}(h_{2'} - h_1) = 15(242 - 232) = 150\ \text{kW}$$

3.
$$Q_{ia} = \dot{m}(h_3 - h_{2'}) = 15(3\,398 - 242) = 47{,}34\ \text{MJ/s}$$

4.
$$P_{ta} = \dot{m}(h_3 - h_{4'}) = 15(3\,398 - 2\,307{,}918) = 16{,}351\ \text{MW}$$

5.
$$Q_{oa} = \dot{m}(h_{4'} - h_1) = 15(2\,307{,}918 - 232) = 31{,}139\ \text{MJ/s}$$

6.
$$P_{oa} = P_{ta} - P_{pa} = 16{,}351 - 0{,}15 = 16{,}201\ \text{MW}$$

7.
$$\eta_{ta} = \frac{P_{oa}}{Q_{ia}} = \frac{16{,}201}{47{,}34} = 0{,}342 = 34{,}223\%$$

Exercise 4.2

1. A steam plant working on the ideal Rankine cycle has a boiler pressure of 500 kPa and a condenser pressure is 60 kPa. Both pressures are absolute values. The feed water enters the pump as a saturated liquid with a density of 1 000 kg/m^3 and a mass flow rate of 20 kg/s The boiler produces steam at a temperature of 300°C.

Calculate the following:

1.1 specific enthalpy at inlet and outlet of the pump

1.2 specific enthalpy at inlet and outlet of the condenser

1.3 power of the pump

1.4 energy transfer rate in the boiler

1.5 power of the turbine

1.6 nett power

1.7 thermal efficiency of the plant

2. A steam plant working on the practical Rankine cycle has a boiler pressure of 2 MPa and a condenser pressure is 32 kPa. Both pressures are absolute values. The feed water enters the pump as a saturated liquid with a density of 1 000 kg/m^3 and a mass flow rate of 10 kg/s. The boiler produces steam at a temperature of 600°C. The pump has an isentropic efficiency of 85% and the turbine has an isentropic efficiency of 92%.

Calculate the following:

2.1 specific enthalpy at inlet and outlet of the pump

2.2 power of the pump

2.3 specific enthalpy at inlet and outlet of the condenser

2.4 energy transfer rate in the boiler

2.5 power of the turbine

2.6 nett power

2.7 thermal efficiency of the plant.

Module summary

- **Sensible heat or liquid enthalpy (h_f):** It is the amount of heat required to change the temperature of water from 0°C

$$h_f = c_w t_w$$

- **Latent heat or enthalpy of evaporation (h_{fg}):** It is the amount of heat required to change the phase from a saturated liquid to a saturated vapour at constant temperature for a unit mass and is measured in J/kg.

- **Superheat or enthalpy of superheat (h_{su}):** It is the amount of heat required to increase the temperature of the saturated steam to a higher value than its saturation temperature for a unit mass and is measured in J/kg

$$h_{su} = h_g + c_{su}(t_{su} - t_s)$$

$$h_g = h_f + h_{fg}$$

Air preheater – It is basically a heat-exchanger installed in the exit flue gas duct of the boiler. The purpose of the air preheater is to recover the heat from the boiler flue gas to increase the temperature of the air that is used for combustion in the furnace.

$$Q_{ap} = m_a C_a(t_{a2} - t_{a1}) \text{ or } Q_{ap} = m_g C_g(t_{4g} - t_{fg})$$

Economiser - It is also known as a feedwater heater. It is a device in which the waste heat of the flue gases is used for heating the feed water before it enters the boiler or evaporator.

$$Q_{ec} = m_s(h_2 - h_1) \text{ or } Q_{ec} = m_g C_g(t_{3g} - t_{4g})$$

Superheater - It is used to dry the wet steam and to increase the temperature of the steam to above its saturation temperature.

$$Q_{su} = m_s(h_4 - h_3) \text{ or } Q_{su} = m_g C_g(t_{2g} - t_{3g})$$

Evaporator – It is also known as the boiler. The evaporator is an essential component of a steam power plant as it is responsible for converting the liquid water into steam that is used to drive the turbines.

$$Q_{ev} = m_s(h_3 - h_2) \text{ or } Q_{ev} = m_g C_g(t_{1g} - t_{2g})$$

Heat energy supplied by fuel

$$Q_f = m_f \times CV$$

Thermal efficiency

$$\eta_t = \frac{Q_s}{Q_f} = \frac{m_s(h_4 - h_1)}{m_f \times CV}$$

Heat balance - The heat balance of a steam plant refers to the systematic accounting of all heat inputs and outputs in a steam generation system.

Formulae for heat balance

a) $Q_f = m_f \times CV$
b) $Q_{su} = m_s(h_4 - h_3)$
c) $Q_{ev} = m_s(h_3 - h_2)$
d) $Q_{ec} = m_s(h_2 - h_1)$

e) $Q_a = m_a C_a (t_{a2} - t_{a1})$
f) $Q_{dg} = m_{dg} C_g (t_g - t_{a1})$
g) $Q_m = m_m (h_{su} - h_f)$
h) $Q_r = a - (b + c + d + e + f + g)$

$m_g = m_a + m_f$

$m_m = 9 \times \%H_2$ (according to combustion)

$m_{dg} = m_a + m_f - m_m$

Rankine cycle - A system that uses the Rankine cycle has four main parts:

Pump: The function of the pump is to isentropically pump water from a low pressure to a high pressure into the boiler.

$W_p = h_2 - h_1$
Or
$W_p = \bar{v}_1 \Delta p$

Boiler: The function of the boiler is to generate steam at constant pressure. The steam is used to drive machinery or turbines that are used to do work.

$q_i = h_3 - h_2$

$q_{ia} = h_3 - h_{2'}$

Turbine: The function of the turbine is to isentropically expand the high pressure superheated steam to a lower condenser pressure over

its vanes to produce work in order to drive a generator that generates electrical power.

$$W_t = h_3 - h_4$$

$$W_{ta} = h_3 - h_{4'}$$

Condenser: The function of the condenser is to remove the heat from the incoming wet steam at constant pressure in order to condensate the steam to a saturated liquid so that it can be pumped back to the boiler.

$$q_o = h_4 - h_1$$

$$q_{oa} = h_{4'} - h_1$$

Changing specific work to work rate (power) (J/s or W)

$$P = \dot{m} \times W$$

Changing specific heat energy to heat energy rate (J/s or W)

$$Q = \dot{m} \times q$$

Isentropic efficiency

$$\eta_{ip} = \frac{h_2 - h_1}{h_{2'} - h_1} = \frac{P_{ps}}{P_{pa}}$$

$$\eta_{it} = \frac{h_3 - h_{4'}}{h_3 - h_4} = \frac{P_{ta}}{P_{ts}}$$

Thermal efficiency

$$\eta_t = \frac{W_o}{q_i} = \frac{W_t - W_p}{q_i}$$

$$\eta_{ta} = \frac{W_{oa}}{q_{ia}} = \frac{W_{ta} - W_{pa}}{q_{ia}}$$

Summative assessment

1. A boiler plant produces 9,5 kg steam per kg fuel at a pressure of 2,4 MPa and a temperature of 275°C. The specific heat capacity of the superheated steam is 2,5 kJ/kgK. The feedwater enters the economiser at 34,6°C and leaves at 113,3°C. The flue gas leaves the economiser at 350°C. The air-fuel ratio is 19:1. Air enters the pre-heater at 19°C and leaves at 165°C. The steam entering the superheater is 5% wet. The calorific value of the fuel is 32,5 MJ/kg. The C_p for air is 1,005 kJ/kgK and C_p for the flue gas is 1,045 kJ/kgK.

Calculate the following:

1.1 heat absorbed by economiser per kg fuel	[3]
1.2 heat absorbed by evaporator per kg fuel	[3]
1.3 heat absorbed by super-heater per kg fuel	[3]
1.4 heat absorbed by air pre-heater per kg fuel	[2]
1.5 temperature of flue gas leaving pre-heater	[2]
1.6 temperature of flue gas leaving superheater	[2]
1.7 heat lost to the flue gas per kg fuel	[3]
1.8 thermal efficiency	[2]

2. A steam boiler plant with a thermal efficiency of 78,75% produces superheated steam at a pressure of 2 MPa and a temperature of 350°C. The fuel has a calorific value of 32 MJ/kg.
The economiser absorbs 2 352 kJ of heat per kg fuel.
The evaporator absorbs 19 586,91 kJ per kg fuel.

The pressure at the chimney base is 100 kPa.
The temperature at the chimney base is 200°C.
The atmospheric temperature is 20°C.
The air-fuel ratio is 18:1
The mass of moisture formed in the flues is 0,6 kg per kg fuel.
The feed water temperature is 32,9°C.
The specific heat capacity for the flue gas is 1,045 kJ/kgK.
The specific heat capacity for the water is 4,2 kJ/kgK.

Calculate the following:

2.1 mass of steam generated per kg fuel [3]
2.2 temperature of water at inlet of evaporator [2]
2.3 steam dryness factor at inlet to superheater [3]
2.4 heat lost to the flue gas per kg fuel [2]
2.5 heat lost to moisture in flue gas per kg fuel [3]
2.6 percentage heat lost due to radiation by setting up a heat balance table in kJ/kg_f [7]

3. A boiler plant consists of an economiser, evaporator and superheater. The economiser absorbs 8% of the heat supplied by the fuel. The superheater absorbs 2 590 kJ per kg fuel. The thermal efficiency of the plant is 80%. The plant uses 1 080 kg of fuel per hour to produce 9 000 kg of steam per hour. The calorific value of the fuel is 30 MJ/kg. Pressure and temperature of the superheated steam is 2 550 kPa and 301°C. The specific heat capacity of the superheated steam is 2,75 kJ/kgK. The heat lost due to moisture in the flue gas is 1 575 kJ per kg fuel and heat lost to the dry products is 1 905 kJ per kg fuel.

Calculate the following:

3.1 power supplied by fuel and specific enthalpy of steam at exit of superheater [3]

3.2 total heat absorbed per second and specific enthalpy of feed water entering economiser [2]

3.3 heat absorbed by economiser per second and specific enthalpy of feed water entering evaporator [2]

3.4 heat absorbed by superheater per second and specific enthalpy of steam leaving evaporator [2]

3.5 dryness factor of steam entering superheater [2]

3.6 heat absorbed by evaporator per second [1]

3.7 percentage heat lost due to radiation by setting up a heat balance table in kJ/s [6]

4. A steam plant working on the ideal Rankine cycle has a boiler pressure of 3 MPa and a condenser pressure is 80 kPa. Both pressures are absolute values. The feed water enters the pump as a saturated liquid with a density of 1 000 kg/m^3. The boiler produces steam at a temperature of 250°C.

Calculate the following:

4.1 specific enthalpy at all the stages [8]

4.2 specific work done by the pump and turbine [2]

4.3 specific heat transfer in the boiler and condenser [2]

4.4 nett work and thermal efficiency of the plant [2]

5. A steam plant working on the ideal Rankine cycle has a boiler pressure of 2 MPa. The boiler produces steam at a temperature of 400°C. The boiler power is 19,799 MW. The feed water enters the pump as a saturated liquid with a density of 995 kg/m^3 and a mass flow rate of 20 kg/s. The nett power is 19,759 MW.

Calculate the following:

5.1 specific enthalpy at exit of turbine [2]

5.2 absolute pressure in the condenser [4]

5.3 dryness fraction of steam at condenser exit [2]

5.4 specific enthalpies at inlet and outlet of pump [2]

5.5. thermal efficiency [2]

6. A steam plant working on the practical Rankine cycle has a boiler pressure of 800 kPa and a condenser pressure is 20 kPa. Both pressures are absolute values. The feed water enters the pump as a saturated liquid with a density of 998 kg/m^3 and a mass flow rate of 15 kg/s. The boiler produces steam at a temperature of 500°C. The pump has an isentropic efficiency of 80% and the turbine has an isentropic efficiency of 90%.

Calculate the following:

6.1 specific enthalpy at inlet and outlet of the pump [3]

6.2 specific enthalpy at inlet and outlet of the condenser [7]

6.3 state the actual condition of steam at outlet of turbine. Motivate your answer. [2]

6.4 nett power [3]

6.5 thermal efficiency of the plant [2]

7. A steam plant working on the practical Rankine cycle has a boiler pressure of 500 kPa and a condenser pressure is 60 kPa. The feed water enters the pump as a saturated liquid with a density of 1 000 kg/m^3 and a mass flow rate of 12 kg/s. The pump has an isentropic efficiency of 80%. The isentropic condition of the steam would have been 98,857% dry at the turbine exit, but due to friction the actual condition is dry saturated.

Calculate the following:

7.1 specific enthalpy at inlet and outlet of the pump [3]

7.2 isentropic and actual specific enthalpy at condenser exit [3]

7.3 temperature and specific enthalpy at boiler exit [4]

7.4 isentropic efficiency of turbine [2]

7.5 nett power [3]

7.6 thermal efficiency of the plant [2]

Total: 120 marks

MODULE

5 *Steam turbines*

By the end of this module, students should be able to:

- draw and label multistage velocity diagrams using a scale;
- determine and calculate the following based on the velocity diagram:
  - angles
  - velocities
  - power
  - diagram efficiency
  - axial thrust; and
- apply the relevant formula pertaining to relative inlet- and outlet velocities at the first and second stages.

Introduction

As its name suggests, a steam turbine is powered by the energy in hot, gaseous steam and works like a cross between a wind turbine and a water turbine. It resembles a wind turbine because it has spinning blades that turn when steam blows past them; and it is like a water turbine because the blades fit snugly inside a sealed outer container so the steam is constrained and forced past them at speed.

5.1 Steam turbines

Steam turbines use high-pressure steam to turn electricity generators at very high speeds, so they rotate much faster than either wind or water turbines. If high-velocity steam is blown onto a curved blade, as shown in Figure 5.1, the steam direction will be changed as it passes across the blade, and will leave in a different direction.
As a result of the change of direction across the blade, the steam will impart a force to the blade due to the change in momentum.

This force will be in the direction shown on the diagram and is known as the radial force. If the blade was free, it would move off in the direction of the radial force.

A second force is developed on the blade, which is perpendicular to the radial force. This force causes end thrust on the shaft and is known as the axial force.

If a number of blades were fixed around the perimeter of a disc and the disc was free to rotate on a shaft, steam blown across the blades would cause the disc to rotate due to the radial force. This is the working principle of the steam turbine.

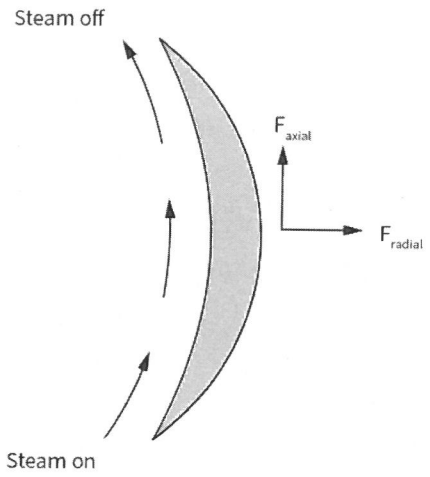


Figure 5.1: Forces on a curved blade

In simple terms, a steam turbine works by using a heat source such as gas, coal, nuclear, or solar to heat water to extremely high temperatures until it is converted into steam. The steam is then accelerated through a nozzle system while decreasing the pressure.

The high velocity steam is directed onto a series of vanes on the circumference of a wheel. As that steam flows over the turbine's spinning blades, the kinetic and thermal energy of the steam is turned into mechanical energy in the rotating turbine's blades.

Because steam turbines generate rotary motion, they are particularly suited for driving electrical generators for electrical power generation. The turbines are connected to a generator with a shaft, which in turn produces energy via a magnetic field that produces an electric current.

Types of steam turbines

Steam turbines may be classified into different categories depending on their construction, working pressures, size, and many other elements. But there are two basic types of steam turbines which are called impulse and reaction turbines. There are other types of steam turbines that are actually derivatives of these two main types.

Impulse turbine

In this type of turbine, the superheated steam is projected at high velocity from fixed nozzles in the casing. When the steam strikes the blades, it causes the turbine shaft to rotate. The high pressure and intermediate pressure stages of a steam turbine are usually impulse turbines. The entire pressure drops of steam take place in stationary nozzles only.

In impulse turbines, the steam expands through the nozzle, where most of the pressure energy is converted to kinetic energy. The high-velocity steam from fixed nozzles impacts the blades changes its direction, which in turn applies a force. The resulting impulse force drives the blades forward, causing the rotor to turn.

The main feature of these turbines is that the pressure drop per single stage can be quite large, allowing for large blades and a smaller number of stages.

The Curtis and Rateau turbines are examples of impulse turbines. Figure 5.2(a) shows cross sectional view of a typical stage in an impulse turbine and Figure 5.2(b) shows the relevant pressure and velocity graphs.

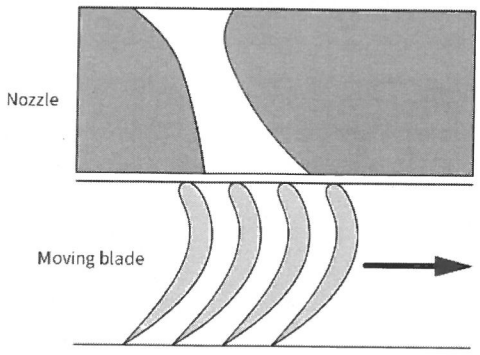


Figure 5.2(a): Impulse turbine

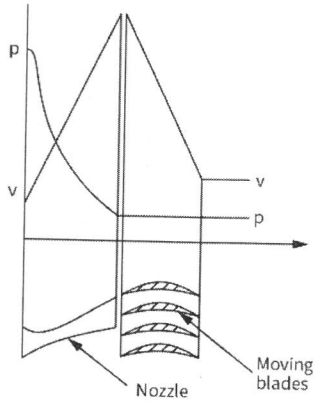


Figure 5.2(b): Graphs

Reaction turbine

In a reaction turbine, the steam expands through the fixed vanes, where the pressure energy is converted to kinetic energy. The high-velocity steam from fixed vanes impacts the blades (which are also shaped like nozzles), changes their direction, and undergoes further expansion.

The change in its direction as well as the steam acceleration applies an extra force. The resulting impulse drives the blades forward, causing the rotor to turn. There is no net change in steam velocity across the stage but there is a decrease in both pressure and temperature.

In this type of turbine, the pressure drops take place in a number of stages, because the pressure drop in a single stage is limited. The main feature of this type of turbine is that in contrast to the impulse turbine, the pressure drop per stage is lower, so the blades become smaller, and the number of stages increases.

On the other hand, reaction turbines are usually more efficient. The reaction turbine was invented by Sir Charles Parsons and is known as

the Parsons turbine. Figure 5.3(a) shows cross sectional view of a typical stage in an impulse turbine and Figure 5.3(b) shows the relevant pressure and velocity graphs.

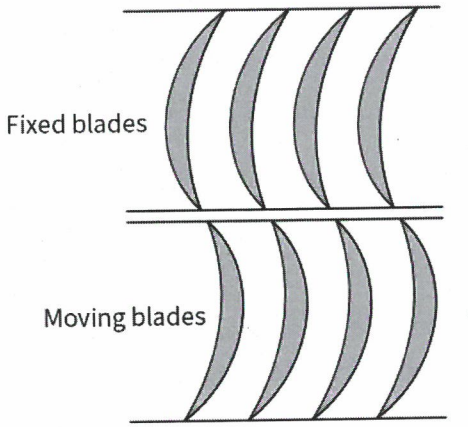


Figure 5.3(a): Reaction turbine

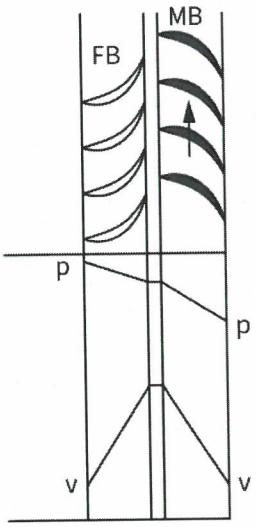


Figure 5.3(b): Graphs

In the case of steam turbines a reaction turbine would require approximately double the number of blade rows as an impulse turbine, for the same degree of thermal energy conversion. Whilst this makes the reaction turbine much longer and heavier, the overall efficiency of a reaction turbine is slightly higher than the equivalent impulse turbine for the same thermal energy conversion.

Modern steam turbines frequently employ both reaction and impulse in the same unit, typically varying the degree of reaction and impulse from the blade root to its edge. The rotor blades are usually designed like an impulse blade at the root and like a reaction blade at the tip.

Since the Curtis turbine reduces the pressure and temperature of the fluid to a moderate level significantly with a high proportion of work per stage, the usual arrangement is to provide one or more Curtis stages on the high-pressure side, followed by Rateau or reaction staging on the low pressure end.

In general, when friction is considered the reaction stage is found to be the most efficient, followed by Rateau and Curtis in that order. Frictional losses are significant for Curtis stages since these are proportional to steam velocity squared.

The reason that frictional losses are less significant in the reaction stage lies in the fact that the steam expands continuously and therefore flow velocities are lower.

In summary, the following table compares impulse and reaction turbines.

Impulse	**Reaction**
1. The steam expands completely through the nozzles only and then remains constant.	1. The steam partly expands in the fixed vanes and partly in the moving vanes.
2. The impulsive force strikes the blades fixed to the rotor.	2. The vector sum of reactive and impulsive forces strikes the blades fixed to the rotor.
3. The blades are symmetrical in shape.	3. The blades are asymmetrical in shape

4. The moving blades of impulse turbines do not have a change in flow area. They do not therefore change the velocity of the steam but only change its direction.	4. The moving blades of reaction turbines do have a change in flow area. They are shaped like nozzles and act to accelerate the steam as it passes through them.
5. The rotational speed is high since the velocity of the steam is high.	5. The rotational speed is much lower than impulse turbine.
6. The number of stages required for producing the same power is less.	6. The number of stages required for producing the same power is more.
7. The blade efficiency curve is higher.	7. The blade efficiency curve is lower.

Table 5.1

Single stage velocity diagrams (revision of N5)

The velocity diagram of a turbine consists mainly out of the following three velocity vectors.

Blade velocity (u)

The blade velocity is the circumferential velocity or linear velocity of the blade due to the rotational frequency of the turbine shaft.

$$u = \pi DN$$

Where:

u is the linear velocity of the moving blade (m/s)

D is the mean diameter of the turbine wheel (m)

N is the rotational frequency of the turbine shaft (rps)

Steam velocity (v)

The steam velocity is the actual velocity of the steam as it enters or leaves the turbine stage. The inlet velocity depends on the change in enthalpy in the nozzle or fixed blade, while the exit velocity depends on the blade angles and friction over the blade.

Relative velocity (v_r)

The relative velocity is the velocity that the steam slides onto or off from the blade.

The relative velocity at inlet depends on the steam velocity, the nozzle angle and the blade velocity.

The relative velocity at exit depends on whether friction is considered over the blade or not.

A friction factor (k) is usually given as a percentage to indicate that friction must be considered.

$$k = \frac{v_{r2}}{v_{r1}}$$

Where:

k is the friction factor (no unit)

v_{r1} is relative velocity at inlet (m/s)

v_{r2} is relative velocity at exit (m/s)

Figure 5.4 shows the velocity diagrams at the inlet and exit of the blade while the blade is moving.

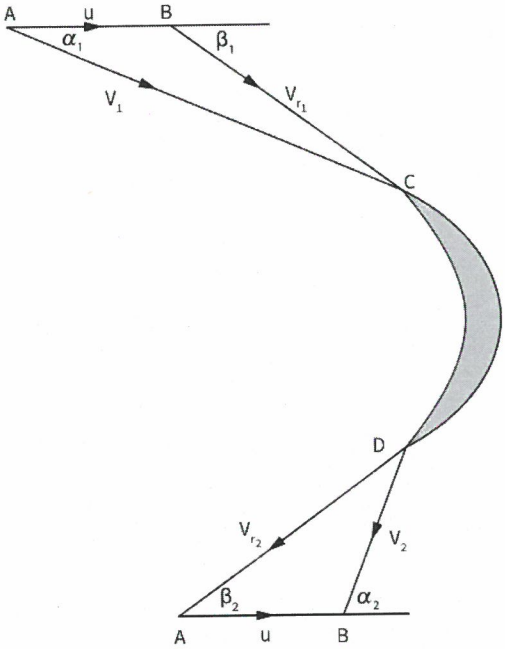


Figure 5.4: Inlet and outlet velocity diagrams on blade

Each diagram can be treated separately in order to do calculations. Figure 5.5(a) shows the velocity diagram at inlet. When accurate calculations need to be done, the value of all the velocities and angles can be calculated using trigonometric ratios.

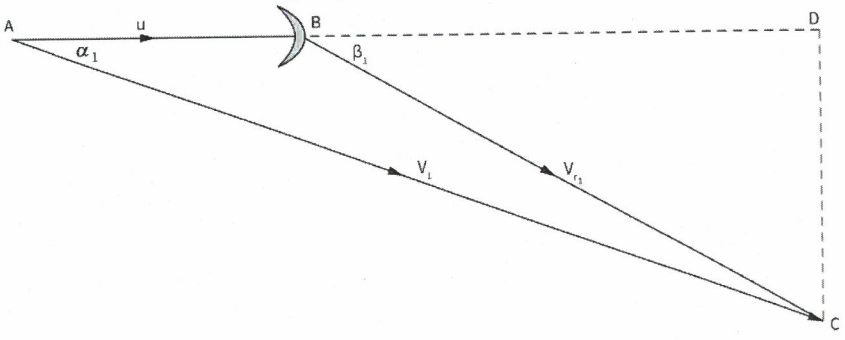


Figure 5.5(a): Inlet velocity triangle of impulse turbine

AB- is velocity of blade (u)
AC- is absolute velocity of steam at inlet (v_1)
BC- is relative velocity at inlet (v_{r1})
AD- is radial component of inlet velocity, also known as whirl velocity at inlet (v_{w1})
CD- is axial component of inlet velocity, also known as flow velocity at inlet (v_{f1}))
α_1- is nozzle angle or inlet angle of steam
β_1- is blade angle at inlet

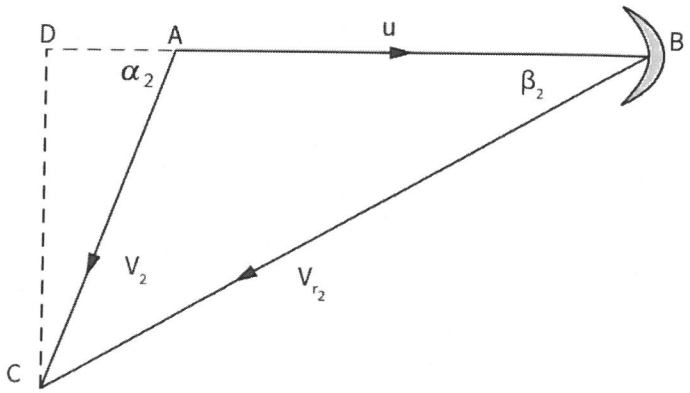


Figure 5.5(b): Exit velocity triangle of impulse turbine

AB- is velocity of blade (u)
AC- is absolute velocity of steam at exit (v_2)
BC- is relative velocity at exit (v_{r2})
AD- is radial component of exit velocity, also known as whirl velocity at exit (v_{w2})
CD- is axial component of exit velocity, also known as flow velocity at exit (v_{f2}))
α_2- is exit angle of steam
β_2- is blade angle at exit

The blade velocity is the mean blade velocity in both diagrams. In order to make it easier to draw the velocity diagram, the inlet and exit diagrams are combined into one diagram by making the distance AB common for both.

Figure 5.6 shows the combined velocity diagrams where ΔABC is the inlet velocity diagram and ΔABD is the exit velocity diagram. These diagrams can be drawn to scale instead of using trigonometry to solve the respective velocities and angles.

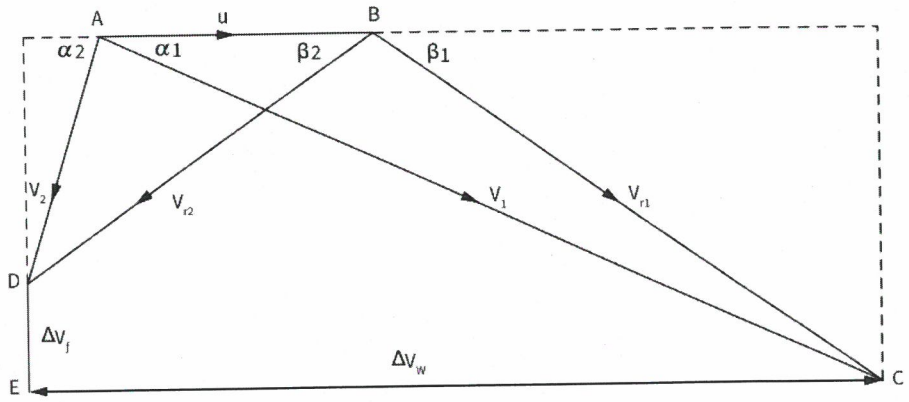


Figure 5.6: Velocity diagram of impulse turbine

EC- is change in whirl velocity (Δv_w)
DE- is change in flow velocity (Δv_f)

$F_R = \dot{m} \times \Delta v_w$
$F_A = \dot{m} \times \Delta v_f$

Where:
F_R is radial force on blade (N)
F_A is axial force or end thrust on shaft (N)
$\dot{m}$ is mass rate of steam (kg/s)

Δv_w is change in whirl velocity (m/s)
Δv_f is change in flow velocity (m/s)

$$P_o = F_R \times u$$

$$P_i = \frac{\dot{m}v_1^2}{2} \text{ (kinetic energy)}$$

$$\eta = \frac{P_o}{P_i}$$

Where:
P_o is output power of turbine (W)
P_i is input power of steam (W)
η is diagram efficiency (percentage)

Example 5.1
Steam is supplied by the nozzle of an impulse turbine at a rate of 5 kg/s. The velocity of the steam from the nozzle is 900 m/s at an angle of 20° to the direction of the blades. The blade velocity is 300 m/s and the blade velocity coefficient is 0,7. The blades are symmetrical. Construct a velocity diagram to a scale of 1 mm = 6 m/s. Indicate the lengths of all the lines as well as the magnitude of the all the angles.

Determine the following from the diagram:
1. blade angles
2. relative velocity at inlet
3. steam velocity at exit
4. calculate the end thrust on the shaft
5. calculate the power developed
6. calculate the diagram efficiency

Solutions

This is an example where the inlet diagram is drawn first.

a) Draw the blade velocity with a length of 50 mm (AB)

b) Draw the steam velocity with a length of 150 mm at an angle of 20° (AC)

c) Complete the inlet diagram by connecting BC

d) Measure BC from the diagram (BC=104,4 mm)

e) Measure blade angle at inlet ($\beta_1 = 29.4°$)

f) Determine length of BD

($BD = 0{,}7 \times BC = 0{,}7 \times 104{,}4 = 73{,}1$ mm

g) Draw line BD at an angle of 29,4° with a length of 73,1 mm

h) Complete the exit diagram by connecting AD

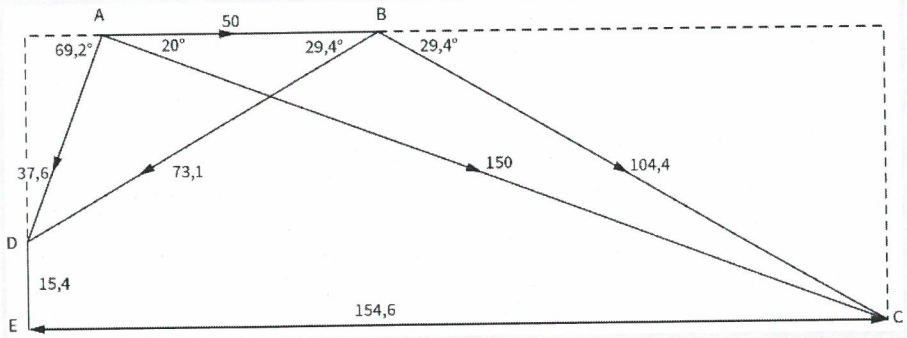


Figure 5.7

(Scale: 1 mm = 6 m/s)

1.

$\beta_1 = 29{,}4° = \beta_2$

2.

$v_{r1} = 104{,}4 \text{ mm} \times 6 = 626{,}4 \text{ m/s}$

3.

$v_2 = 37{,}6 \text{ mm} \times 6 = 225{,}6 \text{ m/s}$

4.

$\Delta v_f = 15{,}4 \text{ mm} \times 6 = 92{,}4 \text{ m/s}$

$F_A = \dot{m} \times \Delta v_f = 5 \times 92{,}4 = 462 \text{ N}$

5.

$\Delta v_w = 154{,}6 \text{ mm} \times 6 = 927{,}6 \text{ m/s}$

$F_R = \dot{m} \times \Delta v_w = 5 \times 927{,}6 = 4\,638 \text{ N}$

6.

$P_o = F_R \times u = 4\,638 \times 300 = 1{,}391 \text{ MW}$

7.

$$P_i = \frac{\dot{m}v_1^2}{2} = \frac{5 \times 900^2}{2} = 2{,}025 \text{ MW}$$

$$\eta = \frac{P_o}{P_i} = \frac{1{,}391}{2{,}025} = 0{,}687 = 68{,}711\%$$

This revision example of N5 was done to get you familiar again to all the different velocities, angles, forces, powers and efficiency formulae.

Note

It is also important to note which of the triangles represent the inlet values and which ones the outlet values.

When two-stage velocity diagrams are now discussed in the next session it is important to understand that the same process is just

repeated for the second stage. There will be two inlet diagrams and two outlet diagrams.

Compounding of steam turbines

The single-impulse turbine is used as a small power machine only. The steam velocities could be as high as 1 070 m/s, and for $\alpha_1 = 20°$ the maximum blade speed would be about 0,47u, giving a maximum blade speed of about 500 m/s.

In practice, the blade speed is limited to about 420 m/s. This velocity used in small machines would give high speeds of rotation of 30 000 r/min. Using smaller diameter rotors mean a more economic construction, but high rotational speeds mean high stresses on the material.

The high turbine speeds are also not directly applicable, and a reduction gear is required to give output speeds that are useful. The expansion of steam in the simple impulse turbine is carried out in a single stage, so the steam velocity at inlet to the wheel is high. There is no drop in pressure in the wheel casing.

The blade velocity should be limited too, for mechanical reasons of strength and operating speed. From these considerations and an inspection of the velocity diagrams, it is evident that the steam leaves the blade wheel with a high absolute velocity.

This generates a loss in the work available on the wheel, although a moderate velocity must be accepted in order to take the steam to the condenser. This leaving loss could amount to 11% of the input energy.

as compounding, are then implemented. Compounding of steam is a method of extracting steam energy in multiple stages.

Pressure compounding (Rateau turbine)

In pressure compounding, the total pressure drop of the steam is divided across multiple stages. Each stage consists of a set of nozzles and moving blades. As steam enters the first set of nozzles, it partially expands, resulting in a decrease in pressure and an increase in velocity. The kinetic energy gained is then absorbed by the moving blades in that stage.

This process is repeated across several stages, allowing for a controlled reduction in pressure while maintaining manageable rotor speeds, which helps mitigate issues like high vibrations and centrifugal forces that can occur at high speeds.

A turbine with a series of simple pressure compounding impulse stages is called a Rateau turbine, shown in Figure 5.8.

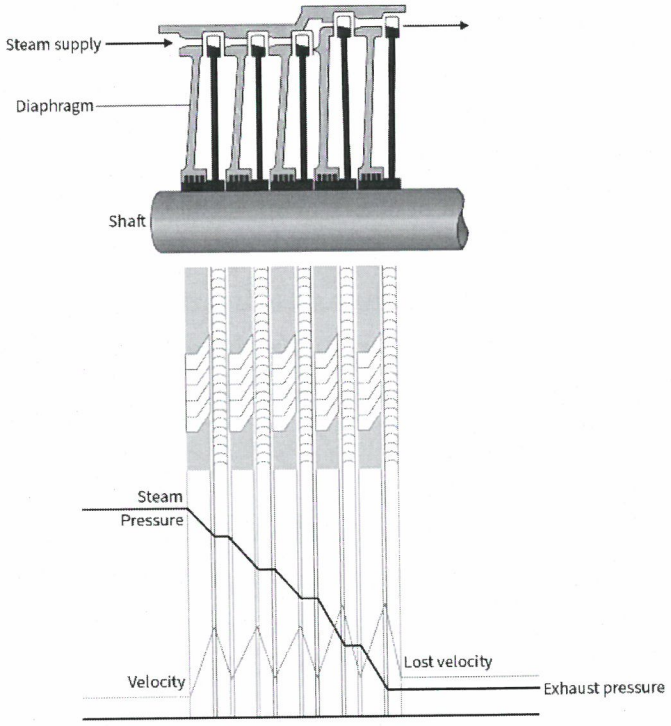


Figure 5.8: Rateau turbine

Velocity compounding (Curtis turbine)

Velocity compounding focuses on managing the steam's velocity rather than its pressure. In this method, a single nozzle produces high-velocity steam, which is then passed over multiple stages of fixed and moving blades.

The high velocity is converted into work across these stages, allowing for a gradual decrease in steam velocity while maintaining a more constant pressure.

This method effectively reduces the turbine's rotational speed and enhances energy extraction from the steam.

A turbine with a series of simple velocity compounding impulse stages is called a Curtis turbine, shown if Figure 5.9.

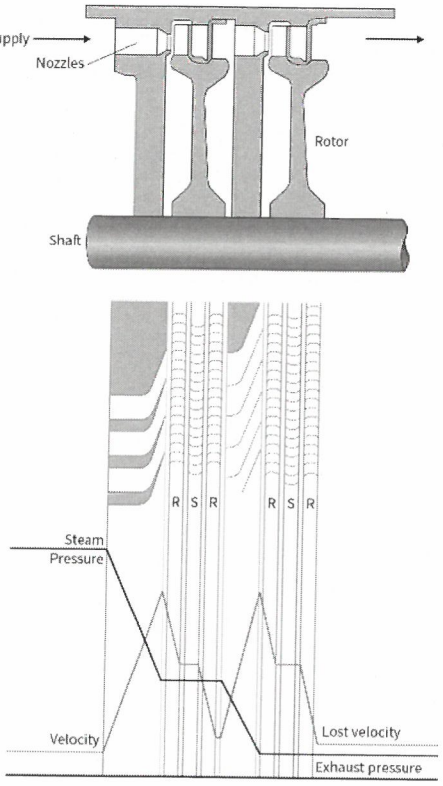


Figure 5.9: Curtis turbine

Two-row velocity diagrams

When considering one of the stages of a compounded turbine, it could consist out of nozzles, two rows of moving blades and one row of fixed blades. Figure 5.10 shows a simplified view of one of these sets of blades with their respective angles.

The purpose of the fixed blade is to change the direction of the steam onto the next row of moving blades. The inlet angle of the fixed vane

is designed to have the same angle as the exit angle of the absolute velocity of the steam leaving the first row. The exit angle of the fixed vane then becomes the inlet angle into the next moving row of blades.

There are three blades where the steam velocity is reduced due to friction.

- First moving blade: $v_{r2} = kv_{r1}$
- Fixed blade: $v_3 = kv_2$:
- Second moving blade: $v_{r4} = kv_{r3}$

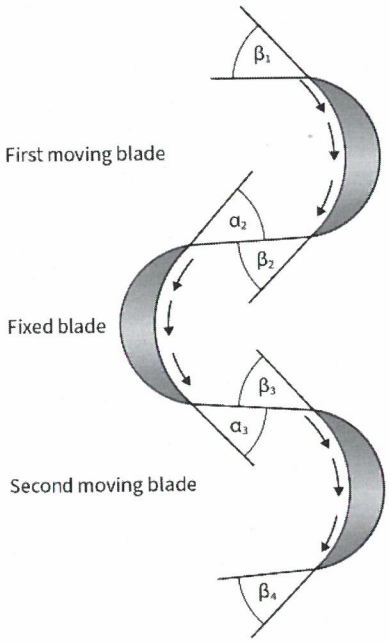


Figure 5.10: Blade angles in a turbine stage

Where
α represents all the absolute steam angles
β represents all the moving blade angles

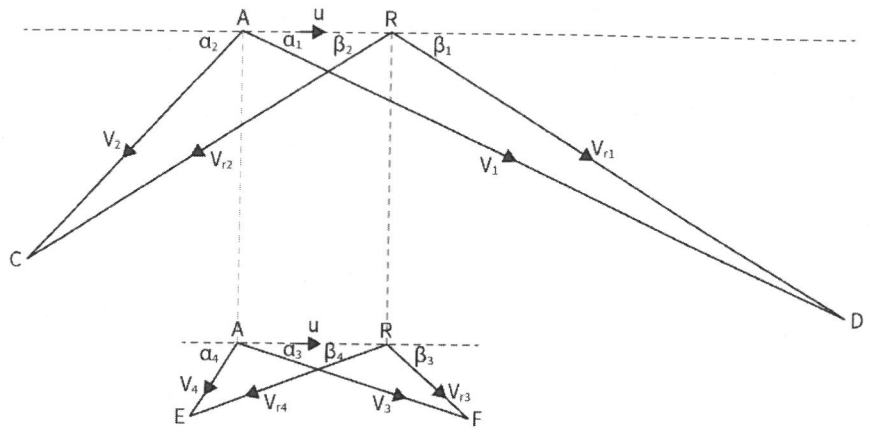


Figure 5.11: Velocity diagrams for two rows

First diagram

ΔARD is inlet triangle of first row (subscript 1)

ΔARC is outlet triangle of first row (subscript 2)

u is average linear blade velocity

α_1 is nozzle angle

v_1 is absolute steam velocity at inlet (nozzle velocity)

α_2 is exit angle of steam from first row (inlet angle of fixed vane)

v_2 is absolute steam velocity at exit from first row

β_1 is blade inlet angle for first row

v_{r1} is relative velocity at inlet for first row

β_2 is blade exit angle for first row

v_{r2} is relative velocity at exit for first row

Each absolute velocity of steam has a horizontal (radial) component, referred to as whirl velocity (v_{w1} and v_{w2})

Each absolute velocity of steam has a vertical (axial) component, referred to as flow velocity (v_{f1} and v_{f2})

Second diagram

ΔARF is inlet triangle of second row (subscript 3)

ΔARE is outlet triangle of second row (subscript 40
u is average linear blade velocity
α_3 is inlet angle for second row (exit angle of fixed blade)
v_3 is absolute steam velocity at inlet for second row
α_4 is exit angle of steam
v_4 is absolute steam velocity at exit from second row
β_3 is blade inlet angle for second row
v_{r3} is relative velocity at inlet for second row
β_4 is blade exit angle for second row
v_{r4} is relative velocity at exit for second row
Each absolute velocity of steam has a horizontal (radial) component, referred to as whirl velocity (v_{w3} and v_{w4})
Each absolute velocity of steam has a vertical (axial) component, referred to as flow velocity (v_{f3} and v_{f4})

Terminology used

- Blades are symmetrical: This means that the inlet and exit angles of the blade are equal ($\beta_1 = \beta_2$) etc.
- No end thrust: The flow velocities are equal ($v_{f1} = v_{f2}$) etc.
- Radial discharge: The exit velocity is radial ($\alpha_4 = 90°$)
- Friction loss is 10% over the blades: ($k = 0,9$)

Tip

When attempting to draw the velocity diagrams, you need to make a rough drawing for yourself so you can see which of the four triangles have the most information. Once that is established, start by drawing that triangle first.

Example 5.2

A velocity compounded impulse turbine consists of two rows of moving blades separated by a row of fixed blades. Steam leaves the nozzles at 540 m/s at a mass rate of 5 kg/s. The discharge angle of the nozzle is 16°. The exit blade angles from the first moving row, the fixed row and the second moving row are 20°, 25° and 28° respectively. The blade speed is 120 m/s. the velocity friction coefficient for all the blades is 0,9.

Construct the velocity diagrams to a scale of 1 mm = 4 m/s. Indicate the lengths of all the lines as well as the magnitude of the all the angles.

Determine the following from the diagram:

1. inlet angle of first moving blade
2. inlet angle of fixed blade
3. inlet angle of second moving blade
4. exit angle of steam
5. calculate the end thrust on the shaft
6. calculate the power developed
7. calculate the diagram efficiency

Solutions

Step 1: Start by drawing triangle 1 first (inlet triangle of 1st row)

Step 2: Complete triangle 2 (outlet triangle of 1st row)

Step 3: Draw triangle 3 (inlet triangle of 2nd row)

Step 4: Complete triangle 4 (outlet triangle of 2nd row)

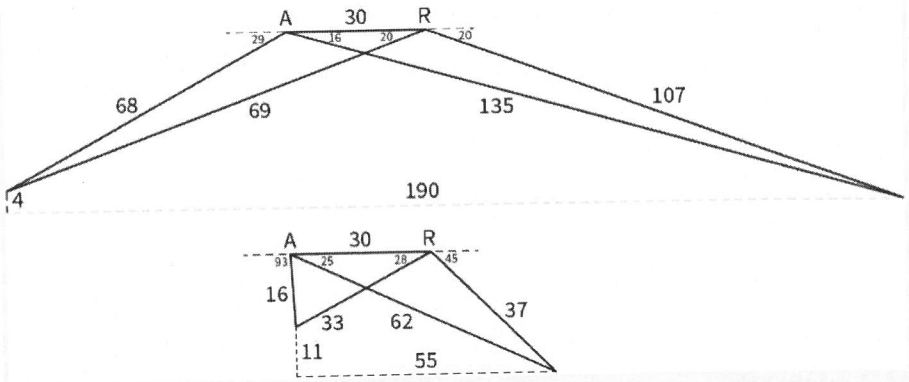


Figure 5.12

(Scale: 1 mm = 4 m/s)

1.

$\beta_1=20^o$

2.

$\alpha_2=29^o$

3.

$\beta_2=45^o$

4.

$\alpha_4=93^o$

5.

$\Delta v_f = (4 + 11)\ \text{mm} \times 4 = 45\ \text{m/s}$

$F_A = \dot{m} \times \Delta v_f = 5 \times 45 = 225\ \text{N}$

6.

$\Delta v_w = (190 + 55)\ \text{mm} \times 4 = 980\ \text{m/s}$

$$F_R = \dot{m} \times \Delta v_w = 5 \times 980 = 4{,}9 \text{ kN}$$

$$P_o = F_R \times u = 4{,}9 \times 10^3 \times 120 = 588 \text{ kW}$$

7.

$$P_i = \frac{\dot{m} v_1^2}{2} = \frac{5 \times 540^2}{2} = 729 \text{ kW}$$

$$\eta = \frac{P_o}{P_i} = \frac{588}{729} = 0{,}807 = 80{,}658\%$$

Example 5.3

A velocity compounded impulse turbine consists of two rows of moving blades separated by a row of fixed blades. The nozzles supply steam with a mass rate of 55 kg/s. The steam enters the fixed vane at 450 m/s. The blade angles of the first moving row are symmetrical at 25° and the blade angles of the second moving row are symmetrical at 30°. The blade speed is 180 m/s. the velocity friction coefficient for all the blades is 0,9.

Construct the velocity diagrams to a scale of 1 mm = 6 m/s. Indicate the lengths of all the lines as well as the magnitude of the all the angles.

Determine the following from the diagram:

1. nozzle angle
2. inlet angle of fixed blade
3. exit angle of fixed blade
4. exit angle of the gas
5. calculate the end thrust on the shaft
6. calculate the power developed
7. calculate the diagram efficiency

Solutions

Step 1: Start by drawing triangle 2 first (outlet triangle of 1st row)

Step 2: Complete triangle 1 (inlet triangle of 1st row)

Step 3: Draw triangle 3 (inlet triangle of 2nd row)

Step 4: Complete triangle 4 (outlet triangle of 2nd row)

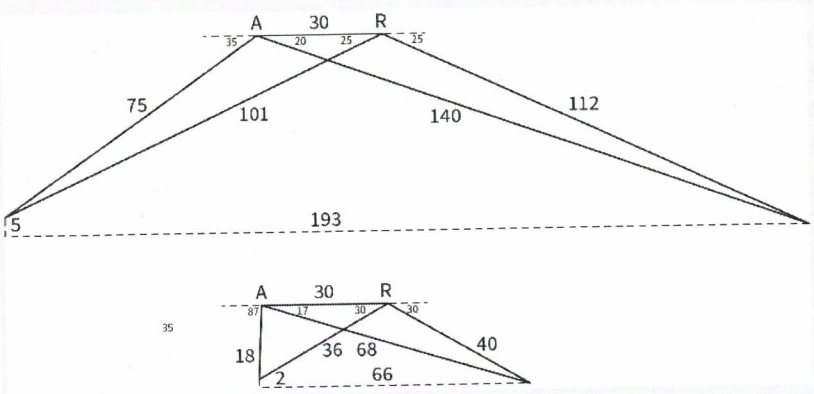


Figure 5.13

(Scale: 1 mm = 6 m/s)

1. $\alpha_1=20^o$

2. $\alpha_2=35^o$

3. $\alpha_3=17^o$

4. $\alpha_4=87^o$

5.

$\Delta v_f = (5 + 2)\ \text{mm} \times 6 = 42\ \text{m/s}$

$F_A = \dot{m} \times \Delta v_f = 55 \times 42 = 2{,}31\ \text{kN}$

6.

$\Delta v_w = (193 + 66)\ \text{mm} \times 6 = 1\,554\ \text{m/s}$

$F_R = \dot{m} \times \Delta v_w = 55 \times 1\,554 = 85{,}47\ \text{kN}$

$P_o = F_R \times u = 85{,}47 \times 10^3 \times 180 = 15{,}385\ \text{MW}$

7.

$v_1 = 140\ \text{mm} \times 6 = 840\ \text{m/s}$

$$P_i = \frac{\dot{m}v_1^2}{2} = \frac{55 \times 840^2}{2} = 19{,}404\ \text{MW}$$

$$\eta = \frac{P_o}{P_i} = \frac{15{,}385}{19{,}404} = 0{,}793 = 79{,}286\%$$

Example 5.4

A velocity compounded impulse turbine consists of two rows of moving blades separated by a row of fixed blades. The nozzles supply steam at a mass rate of 5 kg/s. The blade angles for all the moving blades are 30°. The exit angle of the fixed blade is 20°. The blade speed is 90 m/s. The velocity friction coefficient for all the blades is 0,9.

Construct the velocity diagrams to a scale of 1 mm = 3 m/s. Indicate the lengths of all the lines as well as the magnitude of the all the angles.

Determine the following from the diagram:

1. exit angle of the gas
2. inlet angle of fixed blade
3. nozzle angle
4. calculate the end thrust on the shaft

5. calculate the power developed
6. calculate the diagram efficiency

Solution

Step 1: Start by drawing triangle 3 first (inlet triangle of 2nd row)
Step 2: Complete triangle 4 (outlet triangle of 2nd row)
Step 3: Draw triangle 2 (outlet triangle of 1st row)
Step 4: Complete triangle 1 (inlet triangle of 1st row)

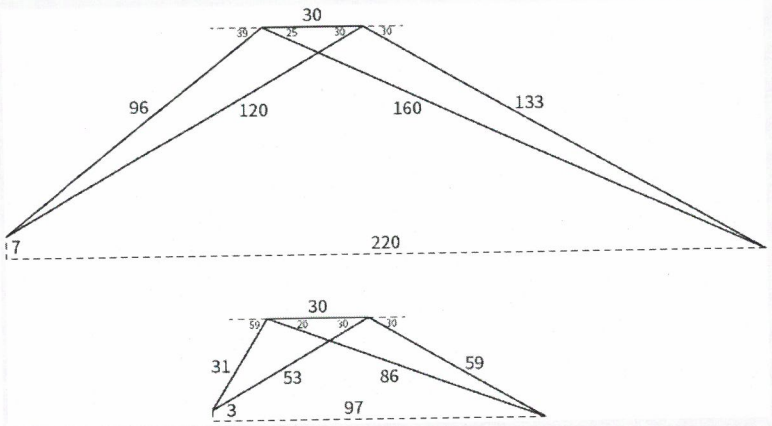


Figure 5.14

(Scale: 1 mm = 3 m/s)

1. $\alpha_4=59^o$

2. $\alpha_2=39^o$

3. $\alpha_1=25^o$

4.

$\Delta v_f = (7 + 3)\ \text{mm} \times 3 = 30\ \text{m/s}$

$F_A = \dot{m} \times \Delta v_f = 5 \times 30 = 150\ \text{N}$

5.

$\Delta v_w = (220 + 97)\ \text{mm} \times 3 = 951\ \text{m/s}$

$F_R = \dot{m} \times \Delta v_w = 5 \times 951 = 4{,}755\ \text{kN}$

$P_o = F_R \times u = 4{,}755 \times 10^3 \times 90 = 427{,}95\ \text{kW}$

6.

$v_1 = 160\ \text{mm} \times 3 = 480\ \text{m/s}$

$$P_i = \frac{\dot{m}v_1^2}{2} = \frac{5 \times 480^2}{2} = 576\ \text{kW}$$

$$\eta = \frac{P_o}{P_i} = \frac{427{,}95}{576} = 0{,}743 = 74{,}297\%$$

Example 5.5

A velocity compounded impulse turbine consists of two rows of moving blades separated by a row of fixed blades. The steam is supplied at 680 m/s by the nozzle. The blade angles for all the moving blades are 30°. There is a 10% loss in velocity across all blades. The final discharge velocity is axial. The power supplied by the nozzles is 2,312 MW.

Construct the velocity diagrams by assuming a length of 30 mm for the blade velocity. Indicate the lengths of all the lines as well as the magnitude of the all the angles.

Determine the following from the diagram:

1. exit angle of fixed blade
2. inlet angle of fixed blade
3. nozzle angle

4. calculate the scale of the diagrams
5. calculate the mass rate of steam supplied
6. calculate the diagram efficiency

Solutions

Step 1: Start by drawing triangle 4 first (outlet triangle of 2nd row)

Step 2: Complete triangle 3 (inlet triangle of 2nd row)

Step 3: Draw triangle 2 (outlet triangle of 1st row)

Step 4: Complete triangle 1 (inlet triangle of 1st row)

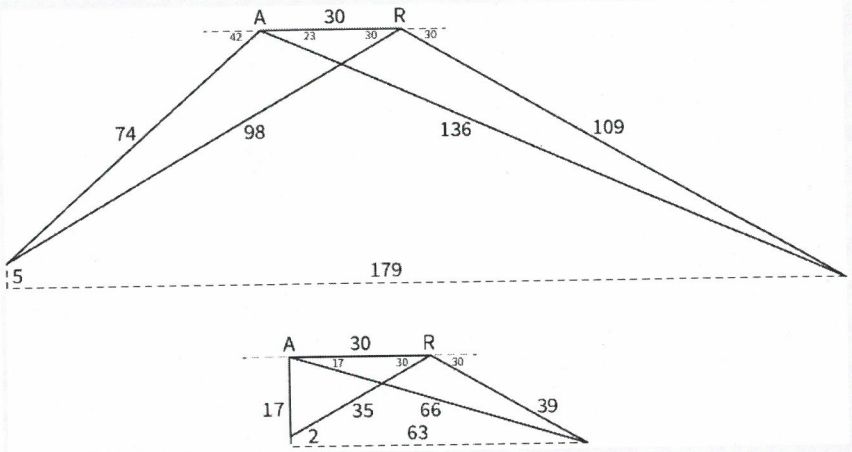


Figure 5.15

(No scale given)

1. $\alpha_3=17^o$
2. $\alpha_2=42^o$
3. $\alpha_1=23^o$

4. Scale: 136 mm = 680 m/s

1 mm = 5 m/s ; scale = 1: 5

5.

$v_1 = 136\ \text{mm} \times 5 = 680\ \text{m/s}$

$$\dot{m} = \frac{2P_i}{v_1^2} = \frac{2 \times 2{,}312 \times 10^6}{680^2} = 10\ \text{kg/s}$$

6.

$\Delta v_w = (179 + 63)\ \text{mm} \times 5 = 1\,210\ \text{m/s}$

$F_R = \dot{m} \times \Delta v_w = 10 \times 1\,210 = 12{,}1\ \text{kN}$

$P_o = F_R \times u = 12{,}1 \times 10^3 \times 150 = 1{,}815\ \text{MW}$

7.

$$\eta = \frac{P_o}{P_i} = \frac{1{,}815}{2{,}312} = 0{,}785 = 78{,}503\%$$

Exercise 5.1

1. Two rows of a velocity-compounded impulse turbine have a mean blade
speed of 170 m/s. Inlet and exit angles of first moving row of blades: 30° and 25° respectively. Inlet and exit angles of the second moving row of blades: 44° and 30° respectively. Gas leaving second row of moving blades is at 90° to the plane of the wheel. 15% loss of velocity in all blades due to friction; 5 400 kg of gas per hour flows through the turbine.

Use a scale of 1 cm = 50 m/s to draw the diagram.

1.1 Find the inlet and exit angles of the fixed blades.
1.2 Find the nozzle angle and velocity of gas leaving the nozzles.
1.3 Find the total thrust of the two rows of blades.
1.4 Find the power developed by the two rows of blades.

2. Data for a two-stage impulse gas turbine: nozzle angle and fixed blade angle: 20° each; velocity of gas leaving nozzles: 500 m/s; blade speed: 100 m/s; velocity coefficient: 0,9.

Use a scale of 3 cm = 100 m/s to draw velocity diagrams for both stages, so that there is no axial thrust developed by the turbine.
2.1 Find the velocity of gas leaving the turbine. (88,5 m/s)
2.2 Find the magnitude and direction of the relative velocity of the gas
at inlet and outlet of the first stage. (410 m/s; 26°) (367 m/s; 29°)
2.3 Find the inlet and outlet angles of the relative velocity of the second stage. (31°; 35°)
2.4 Find the power developed per kilogram of gas per second. (95,3 kW)
2.5 Find the diagram efficiency. (76,3%)

3. Data for a velocity-compounded two-stage impulse gas turbine: velocity
coefficient = 0,95; gas flow = 45 kg per second; moving blade outlet angle of first stage = 25°, and moving blade outlet angle of second stage = 30°. The gas leaves the turbine at 90° to the wheel. The gas leaves the nozzles of the first stage at 635 m/s.

Construct a velocity diagram where no thrust is developed by the turbine.
(Assume the scale for the peripheral speed of the wheel as 4 cm.)

3.1 Find the nozzle angle.
3.2 Find the angle of gas entering the second stage
3.3 Find the blade speed in metres per second.
3.4 Find the direction of the gas leaving the first stage.

3.5 Find the velocity of gas leaving the turbine.
3.6 Find the total whirl velocity.
3.7 Find the power developed by the turbine.

Module summary

Types of turbines

- Impulse: The steam expands completely through the nozzles only and then remains constant.
- Reaction: The steam partly expands in the fixed vanes and partly in the moving vanes.

Steam compounding

Compounding of steam is a method of extracting steam energy in multiple stages.

Types of compounding

- Pressure compounding: The total pressure drop of the steam is divided across multiple stages. The velocity stays more or less constant overall.
- Velocity compounding: The velocity of steam increases and pressure drops in the fixed blades and velocity decreases while pressure remains constant in the moving blades

Formulae

$u = \pi DN$

$k = \frac{v_{r2}}{v_{r1}}$

$F_A = \dot{m} \times \Delta v_f$

$$F_R = \dot{m} \times \Delta v_w$$

$$P_o = F_R \times u$$

$$P_i = \frac{\dot{m} v_1^2}{2}$$

$$\eta = \frac{P_o}{P_i}$$

Summative assessment

1. A velocity compounded impulse turbine consists of two rows of moving blades separated by a row of fixed blades. The average diameter of the turbine rotor is 976 mm and it has a rotational frequency of 3 131 rpm.

- The flow velocity at inlet to the first stage is 225 m/s.
- The velocity of flow at inlet to the second stage is 100 m/s.
- The outlet angle of the first row of moving blades is 25°.
- The outlet angle of the second row of moving blades is 28°.
- The steam leaves the turbine at an angle of 80°.
- There is a 4% loss of velocity across all blades due to friction.

Construct the velocity diagrams to a scale of 1 mm = 5 m/s. Indicate the lengths of all the lines as well as the magnitude of the all the angles. [10]

Determine the following from the diagram:

1.1 nozzle angle [1]

1.2 inlet angle of the fixed blades [1]

1.3 outlet angle of the fixed blades [1]

1.4 inlet angle of the second row of moving blades [1]

1.5 inlet angle of the first row of moving blades [1]

1.6 inlet velocity of the steam	[1]
1.7 velocity of the steam leaving the first stage	[1]
1.8 velocity of the steam leaving the fixed blades	[1]
1.9 velocity of the steam leaving the turbine	[1]
1.10 diagram efficiency	[1]

2. A velocity compounded impulse turbine consists of two rows of moving blades separated by a row of fixed blades.
The first row moving blades is symmetrical with angles of 25°.
The second row moving blades is symmetrical with angles of 30°.
The exit angle of the fixed blades is 17°.
The relative velocity at the outlet of the first stage is 575 m/s.
The mass flow rate is 2 kg/s.
The velocity coefficient for all the blades is 0,96.

2.1 Construct the velocity diagrams by using a length of 36 mm for the average blade velocity. Indicate the lengths of all the lines as well as the magnitude of the all the angles.

Determine the following from the diagram:

2.2 scale of the diagrams	[1]
2.3 steam velocity from nozzle	[1]
2.4 velocity at exit from the first stage	[1]
2.5 velocity at exit of the turbine	[1]
2.6 velocity at entrance to the second stage	[1]
2.7 relative velocity at inlet to first stage	[1]
2.9 angle of the steam leaving turbine	[1]
2.8 power of turbine	[2]
2.9 power supplied by nozzle	[1]

3. A velocity compounded impulse turbine consists of two rows of moving blades separated by a row of fixed blades.
The exit velocity of the nozzle is 640 m/s
The average linear velocity of the blade is 150 m/s.
The first row of moving blades is symmetrical.
The relative velocity at exit from the first row is 475 m/s.
The inlet angle of the second row of moving blades is 30°.
No axial thrust is developed in the second stage.
The mass flow rate of the steam is 50 kg/s.
There is a 5% loss of velocity across all blades.

3.1 Construct the velocity diagrams to a scale of 1 mm = 5 m/s. Indicate the lengths of all the lines as well as the magnitude of the all the angles. [10]

Determine the following from the diagram:

3.2 velocity at exit from the first stage [1]
3.3 velocity at inlet to the second stage [1]
3.4 moving blade angles for first row [1]
3.5 exit angle of steam from the turbine [1]
3.6 axial thrust on shaft [2]
3.7 power developed [2]
3.8 diagram efficiency [2]

4. A velocity compounded impulse turbine consists of two rows of moving blades separated by a row of fixed blades.

- The nozzle angle is 20°.
- The mean blade velocity is 160 m/s
- The discharge is axial.
- The inlet angle for the fixed blade is 36°.
- The outlet angle of the fixed blade is 17°.

- The relative exit velocity from the second row is 180 m/s.
- The mass flow rate of the steam is 40 kg/s.
- There is a 6% loss of velocity across all blades.

4.1 Construct the velocity diagrams to a scale of 1 mm = 4 m/s. Indicate the lengths of all the lines as well as the magnitude of the all the angles. [10]

Determine the following from the diagram:

4.2 inlet angle of first row of moving blades [1]
4.3 exit angle of first row of moving blades [1]
4.4 inlet angle of fixed blades [1]
4.5 exit angle of fixed blades [1]
4.6 inlet velocity of steam [1]
4.7 exit velocity of steam [1]
4.7 radial force on the blade [1]
4.8 power developed [1]
4.9 nozzle power [1]
4.10 diagram efficiency [2]

5. A velocity compounded impulse turbine consists of two rows of moving blades separated by a row of fixed blades.

- The exit velocity of the steam is axial at 100 m/s.
- The relative velocity at exit of the second row is 200 m/s.
- The moving blades are all symmetrical.
- The angle at entry to the fixed blades is 36°.
- The velocity coefficient for all blades is 0,93.
- The mass flow rate is 6 kg/s.

5.1 Construct the velocity diagrams to a scale of 1 mm = 5 m/s. Indicate the lengths of all the lines as well as the magnitude of the all the angles. [10]

Determine the following from the diagram:

5.2 nozzle angle [1]

5.3 outlet angle of fixed blades [1]

5.4 inlet velocity of steam [1]

5.5 velocity of steam leaving first stage [1]

5.6 inlet velocity of steam [1]

5.7 axial force on the shaft [1]

5.8 radial force on the blade [1]

5.9 power developed [1]

5.10 nozzle power [1]

5.11 diagram efficiency [2]

Total: 100 marks

MODULE

6 *Renewable energy systems*

By the end of this module, students should be able to:

- understand major concepts of wind energy, including:
  - wind speed
  - wind turbine available
  - the way wind turbines infrastructure is arranged;
- illustrate the power plant using a schematic diagram;
- calculate the air parameters using appropriate formulae for the following conditions:
  - Betz Law
  - maximum efficiency
- apply the relevant formulae to determine the following:
  - the ideal and actual turbine power considering the Betz limit
  - the coefficient of power C p and maximum coefficient of power
  - the swept area, wind power and theoretical power output considering the cross-sectional area and velocity of the wind turbine;
- understand the concept of hydro-electric systems;
- perform calculations using the appropriate formulae for parameters of hydroelectric power plants; and
- determine the following using the relevant formulae (including theoretical horsepower formulae):
  - flow rate
  - static head
  - power.

Introduction

Renewable energy technologies are playing an increasingly important role in generating electricity worldwide. While wind and hydropower are two of the most prominent renewable electricity generation methods today, the renewable energy landscape is rapidly evolving.

Solar photovoltaic systems, geothermal energy, bioenergy and marine energy are also playing an increasing role in the global electricity mix. Continuous innovation is making these technologies more efficient and cost-effective, accelerating the transition to a sustainable energy future.

6.1 Wind electricity generation

Wind power harnesses the kinetic energy of wind to generate electricity. The basic principle involves using wind to propel blades, which then turn an axle attached to the center of the blades. This axle carries the energy to a gearbox and finally to a generator where electricity is produced.

Some key advantages of wind power include:

- It is a free, sustainable energy resource created as wind blows over the ocean surface
- Wind farms can be built on land or offshore to capture strong, consistent winds
- It is one of the fastest growing renewable energy sources, with costs continuing to decline

6.1.1 Wind energy concepts

Wind energy is a significant and rapidly growing source of renewable energy, harnessing the kinetic energy of wind to generate electricity. Understanding its major concepts involves examining wind speed,

the types of wind turbines available, and the arrangement of wind turbine infrastructure.

Wind speed

Wind speed is a critical factor in wind energy generation. The power available from wind is proportional to the cube of the wind speed, meaning that even small increases in wind speed can lead to significant increases in energy production.
For instance, doubling the wind speed can increase the available power by a factor of eight.

This relationship highlights the importance of site selection for wind farms, as locations with consistently high wind speeds are more likely to yield greater energy outputs.

Wind turbines available

Wind turbines are the primary technology used to convert wind energy into electricity. There are two main types of wind turbines:

Onshore wind turbines

These are located on land and are often found in open fields or coastal areas. They typically have lower installation costs and are easier to maintain compared to offshore turbines.

Offshore wind turbines

Positioned in bodies of water, these turbines can capture stronger and more consistent winds, leading to higher energy production.
However, they are generally more expensive to install and maintain due to their remote locations and the complexities of marine environments.

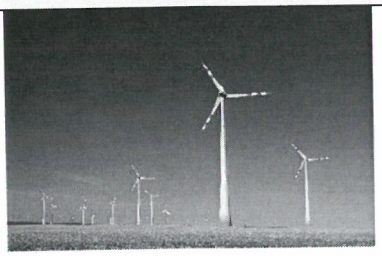
	Offshore turbines	Onshore turbines
	Figure 6.1a	 Figure 6.b
Size	Larger turbines	Smaller turbines
Location	On water bodies	On land
Installation	Complex installation	Less complicated installation
Power Transmission	Through undersea cables	Directly connected to the grid
Maintenance	Difficult to maintain	Easy to maintain

Table 6.1: Differences between offshore and onshore wind turbines

Infrastructure arrangement

The wind turbine infrastructure arrangement refers to the various components and systems that make up a wind farm or wind power plant. When considering the setup of wind farms, the arrangement of wind turbine infrastructure is critical to maximizing efficiency and ensuring safety. Wind farms can be designed in various configurations, depending on the landscape and wind patterns

Here are the key factors involved in the arrangement and installation of wind turbines.

Turbine spacing

Proper spacing between turbines is vital to avoid wind shadow effects, where one turbine blocks wind from reaching another. A common guideline is to space turbines at least 5-10 rotor diameters apart in the

prevailing wind direction (behind each other) and 3-5 rotor diameters apart perpendicular to the wind (next to each other).

Foundation and layout

The installation process begins with laying foundations, which must be designed to support the weight of the turbines. Developers often conduct geotechnical surveys to determine the best foundation design based on soil conditions.

Access roads and maintenance

Roads must be constructed to facilitate access for maintenance and installation. These roads should be designed to accommodate heavy vehicles and equipment used during construction and ongoing maintenance.

Orientation

The most crucial factor in wind farm development is assessing the wind resource at the site. A minimum average wind speed of 11-13 mph is typically required for a viable wind energy facility. Turbines are often oriented to face prevailing winds, which can vary by location and season.

Grid connection

Minimizing the distance to existing transmission infrastructure is essential to reduce costs. Developers should consider the availability of high-voltage lines when selecting a site.

Wind farms are typically connected to the electrical grid through transformers and underground cables, enabling the distribution of generated electricity to homes and businesses. The infrastructure

must also account for energy storage solutions to manage the variable nature of wind energy.

Setback distances

Wind turbines must also be set back a minimum distance from roads, property lines, and other obstructions for safety and to avoid interference with wind flow:

- Blade clearance: The lowest point of the rotation circle of the turbine blades must be at least 9 m above any obstacle.
- Obstacle setback: Turbines should be at least 1,5 times the total turbine height away from the nearest obstacle.
- Property line setback: Many jurisdictions require turbines to be set back from property lines by a distance of 1,1 times the total turbine height.

These setback distances help ensure the safety of nearby structures and people, while also allowing for sufficient wind flow to the turbines. The exact setback requirements vary by location and are specified in local zoning ordinances. In summary, optimizing wind turbine spacing is a balance between maximizing energy production and minimizing interference between turbines.

Installation process

Note the following factors involved in the installation process.

Construction activities

The construction phase involves significant activity, including the installation of met masts for wind measurement, laying foundations, and erecting turbines. This phase can see up to 50 vessels or vehicles working simultaneously in offshore/onshore projects, highlighting the scale of operations.

Safety protocols

During installation, safety zones are established around construction sites to protect workers and vessels. For example, a 500-meter safety zone may be enforced around installation vessels or vehicles during construction.

Operational considerations

Once operational, wind farms require ongoing maintenance, which may involve technicians accessing turbines. Regular inspections and maintenance are essential to ensure reliability and efficiency.

By addressing these elements effectively, developers can optimize the longevity and performance of wind energy projects.

Types of wind turbines

Wind turbines are primarily categorized into two main types based on their axis of rotation: Horizontal Axis Wind Turbines (HAWT) and Vertical Axis Wind Turbines (VAWT).

Horizontal axis wind turbines (HAWT)

Figure 6.2: Horizontal axis wind turbine

HAWTs are the most common type of wind turbine, recognizable by their large blades that resemble airplane propellers. Wind creates lift that causes the blades to rotate. The rotation of the blades powers the drive shaft.

The kinetic energy produced by the turning shaft generates DC electricity which is converted into AC electricity via a transformer and then fed into the grid from the turbine itself. These turbines are typically mounted on tall towers, allowing them to capture stronger winds at higher altitudes.

HAWTs must be oriented to face the wind, which requires a yaw mechanism to adjust their position accordingly.

Examples of HAWTs include:

- Upwind turbines: These turbines have blades that face into the wind, maximizing efficiency.
- Downwind turbines: In this design, the blades are positioned downwind of the tower, which can reduce stress on the structure but may be less efficient overall.

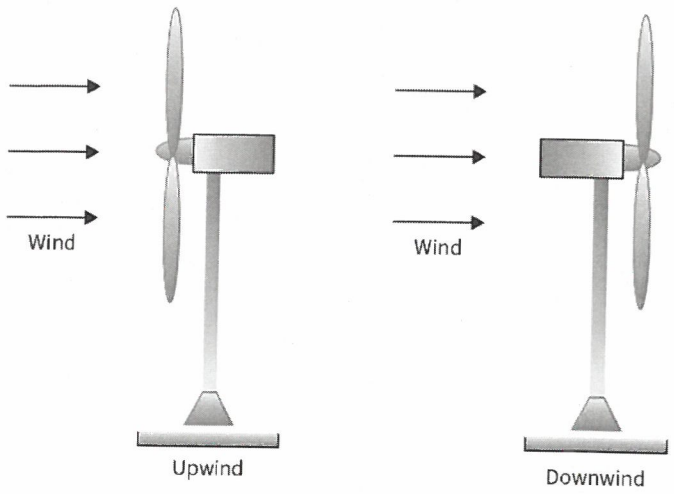


Figure 6.3: Difference between upwind and downwind

Vertical axis wind turbines (VAWT)

VAWTs have blades that are oriented vertically, allowing them to capture wind from any direction without needing to be adjusted. This design makes them suitable for urban environments and smaller installations, as they can be placed closer to the ground, facilitating easier maintenance.

Examples of VAWT include:

- Darrieus turbine: This type features curved blades attached to a vertical rotor, resembling an eggbeater.

Figure 6.4: Darrieus turbine

- Savonius turbine: This design consists of two scoops that catch the wind, making it effective at lower wind speeds but generally less efficient than HAWTs.

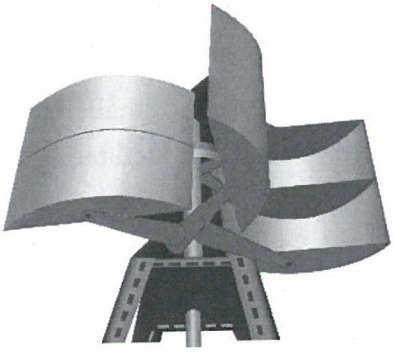


Figure 6.5: Savonius turbine

In summary, HAWTs are preferred for large-scale energy production due to their efficiency, while VAWTs are more suited for restricted energy generation and smaller applications.

Functionality

- Wind energy capture: Wind flows over the rotor blades, creating lift and causing the rotor to spin.
- Energy conversion: The spinning rotor turns the gearbox, which increases the speed of rotation to the generator, producing electricity.
- Voltage regulation: Electricity is transformed to a higher voltage for transmission, reducing losses over long distances.

Components of the wind power plant

These components work together to harness wind energy efficiently, converting it into a reliable source of electricity. The function of each component is discussed in conjunction with Figure 6.7.

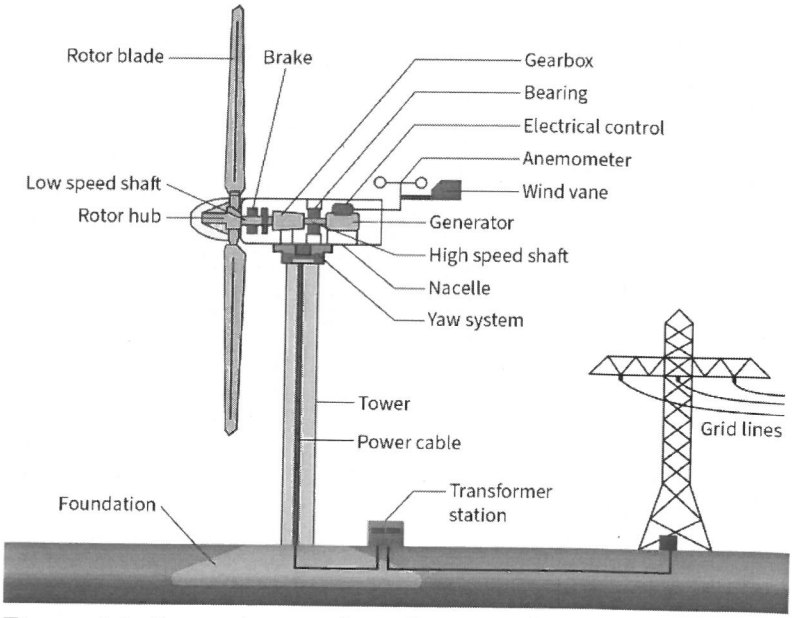


Figure 6.6: Components of wind power plant

Foundation

The foundation provides stability for the wind turbine. For onshore turbines, it is typically a concrete block, while offshore turbines may use floating foundations or structures anchored to the seabed. The foundation must be robust enough to support the turbine's weight and withstand wind forces.

Tower

The tower supports the turbine and elevates the rotor blades to capture stronger winds at higher altitudes. Towers are usually made of tubular steel to reduce drag and can range from a few meters to over 110 meters in height. They must be tall enough to accommodate the rotor's diameter and are usually two to three times the diameter of the blades. Wind speeds can increase dramatically at heights, so taller towers are subjected to stronger winds and therefore generate more electricity.

Rotor blades

The rotor consists of blades designed with an aerodynamic airfoil design to efficiently capture wind energy. Most wind turbines have three blades attached to a central hub. The blades are often hollow and constructed from composite materials to balance strength and weight. The rotor's movement is crucial for converting wind energy into mechanical energy.

Hub

The hub is part of the rotor, securing the three blades and connecting them to the drive shaft in the nacelle. The hub has a cast iron structure weighing between 7 and 14 tons, as it needs to be sturdy enough to support the weight of the turbine blades. Wind turbine hubs have an average length of 4-6 meters, and a diameter of 3-4

meters. Wind turbines are often measured in terms of "hub height," which refers to the distance from the ground to the center of the hub.

Nacelle

Situated at the top of the tower, the nacelle houses the wind turbine's electromechanical system. A nacelle is fixed to a yaw bearing that allows it to revolve with the rotor to match the direction of the wind. Nacelles used on utility-scale wind turbines can contain over 1,500 separate components, be over 15 meters in length, and weigh more than 300 tons.

The nacelle houses essential components, including:

- **Generator**

  Wind turbine generators are situated inside the nacelle and convert the kinetic energy produced by the spinning rotor blades into electrical energy by means of electromagnetic induction.

  Wind turbine generators differ from other generators as they have to cope with a source of torque that is constantly fluctuating. Because of the amount of power they produce, utility-scale wind turbine generators require cooling systems to keep them from overheating.

  Most often the generator is situated near a duct that uses a fan to provide cooling, but some generators are fitted with water cooling systems.

- **Gearbox**

  The gearbox helps to convert the relatively slow rotational speed of the turbine rotor to the high speeds needed to run the generator. The rotor achieves around 18 rpm, while the

generator will need around 1 800 rpm to successfully generate electricity.

- **Drive shaft**
  The turbine's drive shafts are integral parts of the drivetrain, working in conjunction with the gearbox to increase rpm and run the generator. The low-speed shaft (also known as the main shaft) connects the hub to the gearbox. A smaller, high-speed shaft connects the gearbox to the generator.

- **Control systems**
  - **Yaw system**
    The yaw system underneath the nacelle is responsible for orienting the turbine according to wind direction (determined by the wind vane and anemometer). The yaw drive and yaw motor work together to adjust the nacelle so it captures wind power as effectively as possible by facing the wind. This system is crucial for maximizing the power of the turbine and keeping the generator running at a more constant rate.
  - **Pitch control**
    Horizontal-axis wind turbines are fitted with hydraulic or electrical control systems that continuously adjust blade pitch to prevent the rotor speed from exceeding maximum capacity in strong winds. To prevent damage, the majority of modern wind turbines will stop rotating if the wind speed goes beyond 88,5 kilometers per hour. The movement of the rotor blades is known as blade pitch.

- **Electrical systems**
  - Transformer: Steps up the voltage of the electricity generated for efficient transmission.
  - Transmission lines: Carry electricity from the wind farm to the grid or end-users.

- **Additional components**
  - Braking system: Ensures the rotor does not exceed safe speeds during high winds.
  - Anemometer and wind vane: Measure wind speed and direction, providing data for the control systems to optimize turbine performance.

Land requirement per GW unit

Although wind power is a very clean alternative to power generation, the land requirements for a wind farm is very big compared to other forms of power plants. The table shows the comparison of the land required for the production of 1 GW of power between wind, solar and nuclear systems.

Energy source	Land required per 1 000 MW	Average MW/km^2
Nuclear	3,4 km^2	295
Solar	15 - 21 km^2	56
Wind	556 km^2	1,8

Table 6.2

Advantages of wind energy:

- Wind energy is a free.
- Very clean, non-polluting form of energy
- Renewable resource.

- No complex maintenance necessary
- Low operation cost.
- Creation of numerous jobs
- Wind turbines can be located on land that is also used for grazing or even farming.

Disadvantages of wind energy:

- A higher initial investment.
- Cost of the machinery is high.
- Visual landscape harm
- High noise produced by the rotor blades.
- Negative impact on birds and wildlife
- Unsteady wind
- Good wind sites are often located in remote locations far from load centre.

6.1.2 Power calculations

The power carried by the wind depends on its mass rate and speed. The basic formula for kinetic energy is used to determine the power of the wind. If the mass is used (kg) in the energy formula, the unit is in joule (J). If the mass rate is used instead (kg/s), the unit is in watt (J/s).

$$P_w = \frac{\dot{m}v^2}{2}$$

Where
P_w is wind power (W)
$\dot{m}$ is mass rate of the air (kg/s)
v is wind speed (m/s)

The wind speed depends on the roughness of the terrain and height of the hub.

$$\frac{v}{v_o} = \left(\frac{H}{H_o}\right)^{\alpha}$$

Where
α is friction coefficient for the terrain (no unit)
Values for friction coefficient varies between 0,1 for smooth ground to 0,4 for cities with high buildings and trees.

The mass rate depends on the density and the flow rate of the air.

$$\dot{m} = \rho A v$$

Where
ρ is density of air (kg/m^3)
A is the swept area of turbine rotor (m^2)

The density of air is influenced by temperature, pressure and humidity.

- Warmer air is less dense than cooler air
- Density decreases with altitude due to less pressure
- Moist air is less dense than dry air

The density of air is 1,225 kg/m^3 at sea level (101,325 kPa) and 15°C (288 K). This value will be used in the calculations, unless it is stated otherwise.

The swept area of the rotor blade depends on its radius

$$A = \pi r^2$$

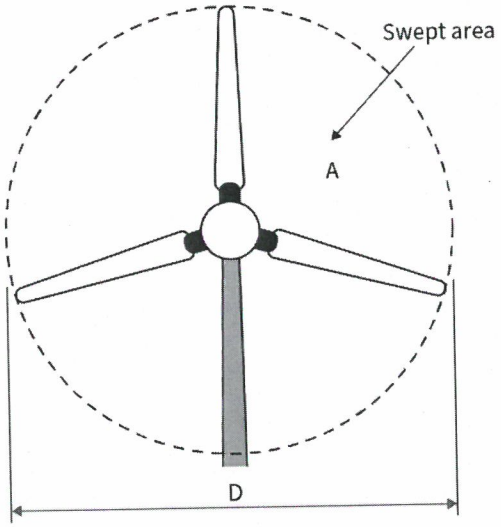


Figure 6.7: Swept area of rotor

Taking all these factors into consideration, the wind power is

$$P_w = \frac{(\rho A v)v^2}{2} = \frac{\rho A v^3}{2}$$

Example 6.1

The wind has a speed of 5 m/s at a height of 10 m over an area with crops and bushes, resulting in a friction coefficient of 0,2. The hub of the horizontal axis turbine used is 80 m above the ground. The density of air is 1,225 kg/m^3.

Calculate the following:

1. The specific wind power (for 1 m^2 swept area) at the 10 m height
2. The specific wind power at the hub height of the turbine
3. The ratio between the powers

Solutions

1.

$$P_{w1} = \frac{\rho A v^3}{2} = \frac{1{,}225 \times 1 \times 5^3}{2} = 76{,}563\ W/m^2$$

2.

$$v = v_o \times \left(\frac{H}{H_o}\right)^{\alpha} = 5 \times \left(\frac{80}{10}\right)^{0{,}2} = 7{,}579\ m/s$$

$$P_{w2} = \frac{\rho A v^3}{2} = \frac{1{,}225 \times 1 \times 7{,}579^3}{2} = 266{,}606\ W/m^2$$

3.

$$P_r = \frac{P_{w2}}{P_{w1}} = \frac{266{,}606}{76{,}563} = 3{,}482$$

Example 6.2

The rotor of a horizontal axis wind turbine has a radius of 20 m. The speed and density of the air in front of the turbine have values of 10 m/s and 1,4 kg/m^3. The speed and density of the air after it flowed through the turbine blades have values of 7 m/s and 1,8 kg/m^3.

Calculate the following:

1. The power of the wind before the turbine
2. The power of the wind after the turbine
3. The power absorbed by the rotor blades
4. The efficiency of the rotor

Solutions

1.

$$A = \pi r^2 = \pi \times 20^2 = 1{,}257 \times 10^3 \text{ m}^2$$

$$P_{w1} = \frac{\rho A v^3}{2} = \frac{1{,}4 \times 1{,}257 \times 10^3 \times 10^3}{2} = 879{,}646 \text{ kW}$$

2.

$$P_{w2} = \frac{\rho A v^3}{2} = \frac{1{,}8 \times 1{,}257 \times 10^3 \times 7^3}{2} = 387{,}924 \text{ kW}$$

3.

$$P_r = P_{w1} - P_{w2} = 1{,}52 \times 10^6 - 387{,}924 \times 10^3 = 491{,}722 \text{ kW}$$

4.

$$\eta_r = \frac{P_r}{P_{w1}} = \frac{491{,}722}{879{,}646} = 0{,}559 = 55{,}9\%$$

When considering the overall efficiency of a wind turbine, several key efficiencies should be considered, including mechanical, electrical and aerodynamic efficiencies.

A breakdown of these efficiencies relevant to wind turbine performance are:

Mechanical efficiency (η_m)

Mechanical efficiency refers to the effectiveness with which the turbine converts the rotational energy from the rotor into usable mechanical energy. This efficiency can be affected by friction in the gearbox, losses in the bearings, and other mechanical components. Mechanical efficiencies can range from 80% to 95%.

Electrical efficiency (η_e)

Electrical efficiency measures how effectively the mechanical energy is converted into electrical energy by the generator. It includes losses due to heat in the generator, inefficiencies in power electronics, and conversion losses when transforming the generated electricity to match grid requirements. Electrical efficiencies can range from 90% to 95%.

Aerodynamic efficiency (η_a or C_p)

Aerodynamic efficiency, better known as power coefficient, relates to how effectively the rotor blades convert the kinetic energy of the wind into mechanical energy. This efficiency is influenced by the design of the blades, the angle of attack, and the wind speed. Aerodynamic efficiencies can range from 35% to 60%.

Overall system efficiency (η_o)

The overall efficiency combines the effects of mechanical, electrical, and aerodynamic efficiencies. It represents the fraction of the wind's kinetic energy that is converted into electrical energy. The overall efficiency of a wind turbine can vary widely but generally falls between 20% and 50%, with peak efficiencies occurring under optimal wind conditions.

$$\eta_o = \eta_m \times \eta_e \times C_p$$

This then means that the power developed by the rotor of a wind turbine depends on the power coefficient and the output power developed by the turbine depends on the overall efficiency.

$$P_r = \frac{C_p \times \rho A v^3}{2}$$

$$P_o = \frac{\eta_o \times \rho A v^3}{2}$$

Wind speed

The wind speed is the most critical factor that affects the power output of a wind turbine, as it is raised to the third power in the formula. This means that a small change in wind speed can result in a significant change in power output.

Swept area

The swept area is the area covered by the rotor blades as they spin. The larger the swept area, the more wind energy the turbine can capture and convert into power. The swept area is proportional to the square of the rotor diameter, so increasing the diameter by a small amount can also increase the power output by a large amount.

However, larger rotors also require stronger and heavier materials, more space, and higher costs, so you need to balance the benefits and drawbacks of different sizes.

Power coefficient

The power coefficient is a measure of how well the turbine converts wind energy into power. It depends on the design and performance of the turbine, such as the shape, number, and pitch of the blades, the gearbox, the generator, and the controller. The power coefficient also changes with the tip speed ratio, which is the ratio of the blade tip speed to the wind speed. The optimal tip speed ratio depends on the type of turbine and can be determined by testing or simulation.

$$\lambda = \frac{v_t}{v} = \frac{4\pi}{n}$$

Where:

λ is tip speed ratio (no unit)

v_t is velocity of blade tip (m/s)

n is number of blades
C_p is power coefficient (no unit)
C_T is torque coefficient (no unit)

Once the tip speed ratio is calculated, the C_p value for a specific turbine can be read off from graphs similar to Figure 6.9.

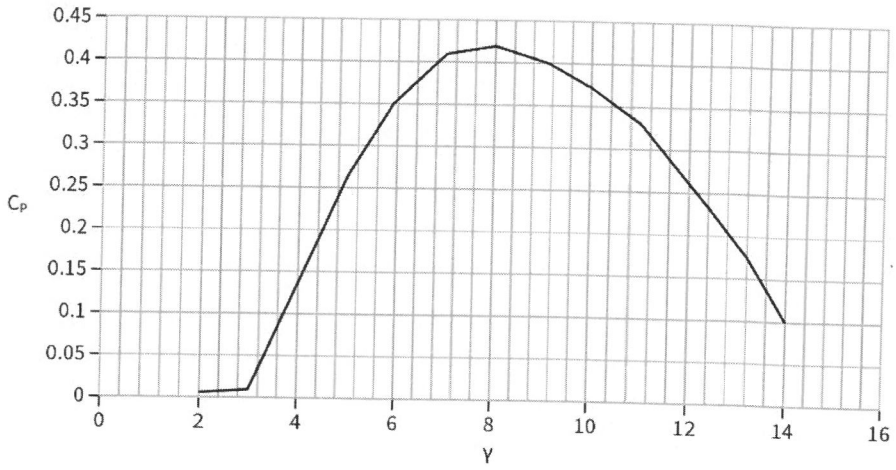


Figure 6.8: C_p vs λ curve

Betz's law

Betz's law, formulated by German physicist Albert Betz in 1919, states that no wind turbine can capture more than 59,3% of the kinetic energy in wind.
This theoretical limit is known as the Betz coefficient. Betz considered the change in momentum of the air flowing through the turbine.

By applying the principles of conservation of mass and momentum, he came to the following conclusion.

At the Betz limit:

- The ratio of outgoing to incoming wind speed is 1/3
- The area of the outgoing wind is 9/4 times the turbine area
- The turbine extracts 16/27 of the kinetic energy from the wind

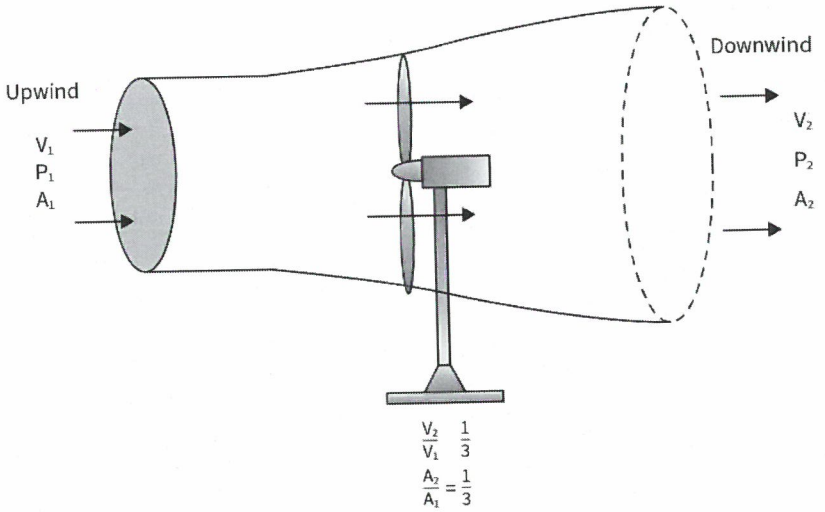


Figure 6.9: Ratios according to Betz

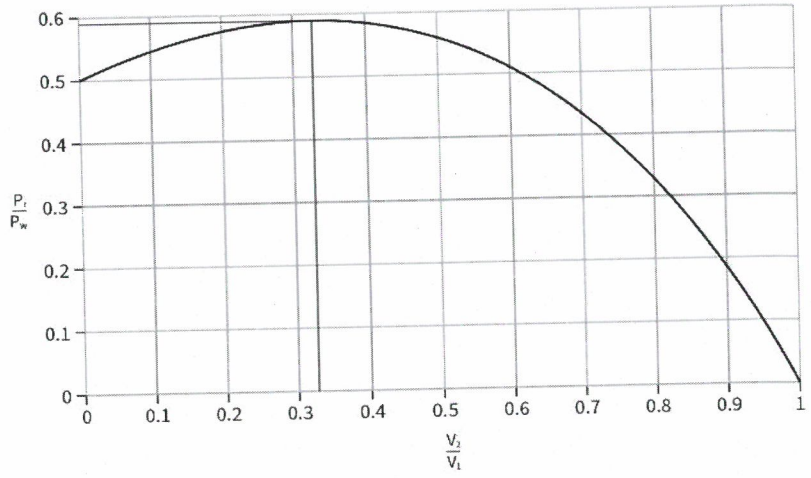


Figure 6.10: Power ratio curve according to Betz

This means that no wind turbine can capture more than 16/27 or 59,3% of the kinetic energy in wind. The Betz limit is derived by comparing the power extracted by the turbine to the available power in the wind upstream and can be presented as

$$C_p = \frac{16}{27} = 0{,}593$$

Example 6.3

The rotor of a horizontal wind turbine has a length of 10 m. The wind speed is 18 km/h. The density of the air is 1,2 kg/m^3. The power coefficient is 0,2.

Calculate the following:

1. The power of the wind
2. The rotor power

Solution

1.

$$A = \pi r^2 = \pi \times 10^2 = 314{,}159 \text{ m}^2$$

$$P_w = \frac{\rho A v^3}{2} = \frac{1{,}2 \times 314{,}159 \times 5^3}{2} = 23{,}562 \text{ kW}$$

2.

$$P_r = C_p \times P_w = 0{,}2 \times 23{,}562 = 4{,}712 \text{ kW}$$

Example 6.4

The rotor of a horizontal axis wind turbine has a length of 25 m and rotates at a speed of 24 rpm. The wind speed is 36 km/h. Assume C_p as 0,22 and the density as 1,225 kg/m^3.

Calculate the following:

1. The number of blades required
2. The rotor power
3. The maximum rotor power possible

Solutions

1.

$$v_t = \pi DN = \pi \times 50 \times 0{,}4 = 62{,}832 \text{ m/s}$$

$$\lambda = \frac{v_t}{v} = \frac{62{,}832}{10} = 6{,}283$$

$$n = \frac{4\pi}{\lambda} = \frac{4\pi}{6{,}283} = 2$$

2.

$$A = \pi r^2 = \pi \times 25^2 = 1{,}963 \times 10^3 \text{ m}^2$$

$$P_r = \frac{C_p \times \rho A v^3}{2} = \frac{0{,}2 \times 1{,}225 \times 1{,}963 \times 10^3 \times 10^3}{2} = 240{,}528 \text{ kW}$$

3.

$$P_{rm} = \frac{C_p \times \rho A v^3}{2} = \frac{0{,}593 \times 1{,}225 \times 1{,}963 \times 10^3 \times 10^3}{2} = 713{,}166 \text{ kW}$$

Example 6.5

The rotor blade of a horizontal axis wind turbine has a length of 50 m. The wind speed is 12 m/s. Assume C_p as 0,4 and the density as 1,3 kg/m^3.

Calculate the following:

1. The rotor power
2. The blade length of the rotor if the power must be increased by 50%
3. The percentage increase in length

Solution

1.

$$A = \pi r^2 = \pi \times 50^2 = 7{,}854 \times 10^3 \text{ m}^2$$

$$P_r = \frac{C_p \times \rho A v^3}{2} = \frac{0{,}4 \times 1{,}3 \times 7{,}854 \times 10^3 \times 12^3}{2} = 3{,}529 \text{ MW}$$

2.

$$P_r = 1{,}5 \times 3{,}529 = 5{,}293 \text{ MW}$$

$$A = \frac{2 \times P_r}{C_p \times \rho v^3} = \frac{2 \times 5{,}293 \times 10^6}{0{,}4 \times 1{,}3 \times 12^3} = 11{,}781 \times 10^3 \text{ m}^2$$

$$r_n = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{11{,}781 \times 10^3}{\pi}} = 61{,}237 \text{ m}$$

3.

$$\% = \frac{\Delta r}{r} = \frac{61{,}237 - 50}{50} = 0{,}225 = 22{,}474\%$$

Example 6.6

The rotor blade of a horizontal axis wind turbine has a length of 30 m. The wind speed is 7 m/s. Assume C_p as 0,3 and the density as 1,2 kg/m^3.

Calculate the following:

1. The rotor power
2. The wind speed if the power needs to be doubled
3. The percentage increase in wind speed

Solution

1.

$$A = \pi r^2 = \pi \times 30^2 = 2{,}827 \times 10^3 \text{ m}^2$$

$$P_r = \frac{C_p \times \rho A v^3}{2} = \frac{0{,}3 \times 1{,}2 \times 2{,}827 \times 10^3 \times 7^3}{2} = 174{,}566 \text{ kW}$$

2.

$$P_r = 2 \times 174{,}566 = 349{,}131 \text{ kW}$$

$$v = \sqrt[3]{\frac{2 \times P_r}{C_p \times \rho A}} = \sqrt[3]{\frac{2 \times 349{,}131 \times 10^3}{0{,}4 \times 1{,}3 \times 2{,}827 \times 10^3}} = 7{,}802 \text{ m/s}$$

3.

$$\% = \frac{\Delta v}{v} = \frac{7{,}802 - 7}{7} = 0{,}115 = 11{,}458\%$$

Example 6.7

The rotor blade of a horizontal axis wind turbine has a length of 25 m. The wind speed is 9 m/s. Assume the density as 1,2 kg/m^3. The output power of the turbine is 252 kW. Mechanical efficiency is 80% and electrical efficiency is 90%.

Calculate the following:

1. The rotor power
2. The power coefficient
3. The overall efficiency

Solutions

1.

$$P_r = \frac{P_o}{\eta_m \times \eta_e} = \frac{252}{0{,}8 \times 0{,}9} = 350 \text{ kW}$$

2.

$$A = \pi r^2 = \pi \times 25^2 = 1{,}963 \times 10^3 \text{ m}^2$$

$$P_w = \frac{\rho A v^3}{2} = \frac{1{,}2 \times 1{,}963 \times 10^3 \times 9^3}{2} = 858{,}833 \text{ kW}$$

$$C_p = \frac{P_r}{P_w} = \frac{350}{858{,}833} = 0{,}408$$

3.

$$\eta_o = \frac{P_o}{P_w} = \frac{252}{858{,}833} = 0{,}293 = 29{,}342\%$$

OR

$$\eta_o = \eta_m \times \eta_e \times C_p = 0{,}8 \times 0{,}9 \times 0{,}408 = 29{,}342\%$$

Exercise 6.1

1. The wind has a speed of 7 m/s at a height of 10 m over an area with smooth hard ground, resulting in a friction coefficient of 0,1. The hub of the horizontal axis turbine used is 100 m above the ground. The density of air is 1,2 kg/m^3.

Calculate the following:

1.1 The specific wind power (for 1 m^2 swept area) at the 10 m height

1.2 The specific wind power at the hub height of the turbine

1.3 The ratio between the powers

2. The rotor of a horizontal axis wind turbine has a radius of 30 m. The speed and density of the air in front of the turbine have values of 9 m/s and 1,3 kg/m^3. The speed and density of the air after it flowed through the turbine blades have values of 6 m/s and 1,6 kg/m^3.

Calculate the following:
2.1 The power of the wind before the turbine
2.2 The power of the wind after the turbine
2.3 The efficiency of the rotor

3. The rotor of a horizontal wind turbine has a length of 15 m. The wind speed is 24 km/h. The density of the air is 1,2 kg/m^3. The power coefficient is 67% of the Betz coefficient.

Calculate the following:
3.1 The power of the wind
3.2 The rotor power

4. The rotor of a horizontal axis wind turbine has a length of 20 m and rotates at a speed of 9 rpm. The wind speed is 14 km/h. Assume C_p as 0,3 and the density as 1,23 kg/m^3.

Calculate the following:
4.1 The number of blades required
4.2 The rotor power
4.3 The maximum rotor power possible

5. The rotor blade of a horizontal axis wind turbine has a length of 45 m. The wind speed is 13 m/s. Assume C_p as 0,4 and the density as 1,3 kg/m^3.

Calculate the following:
5.1 The rotor power
5.2 The blade length of the rotor if the power must be increased by 30%
5.3 The percentage increase in length

6. The rotor blade of a horizontal axis wind turbine has a length of 40 m. The wind speed is 7 m/s. Assume C_p as 0,3 and the density as 1,2 kg/m^3.

Calculate the following:
6.1 The rotor power
6.2 The wind speed if the power is decreased by 50%
6.3 The percentage decrease in wind speed

7. The rotor blade of a horizontal axis wind turbine has a length of 30 m. The wind speed is 9 m/s. Assume the density as 1,2 kg/m^3. The output power of the turbine is 310 kW. Mechanical efficiency is 85% and electrical efficiency is 92%.

Calculate the following:
7.1 The rotor power
7.2 The power coefficient
7.3 The overall efficiency

6.2 Hydroelectric power generation

Hydropower is the oldest and largest source of renewable electricity generation today. Hydroelectric power generation is a method of producing electricity by harnessing the energy of moving water.

This renewable energy source utilizes the potential energy of water stored at higher elevations, which is converted into kinetic energy as it flows down through turbines.
In a hydroelectric scheme, water is stored in a dam and passed through a turbine and generator set before being released back into the river downstream.

6.2.1 Hydroelectric system concepts

The process begins with water being collected or stored in a reservoir, typically created by a dam. When water is released from the reservoir, it flows through large pipes known as penstocks, driving turbines that convert the kinetic energy of the moving water into mechanical energy. This mechanical energy is then transformed into electrical energy by generators connected to the turbines.

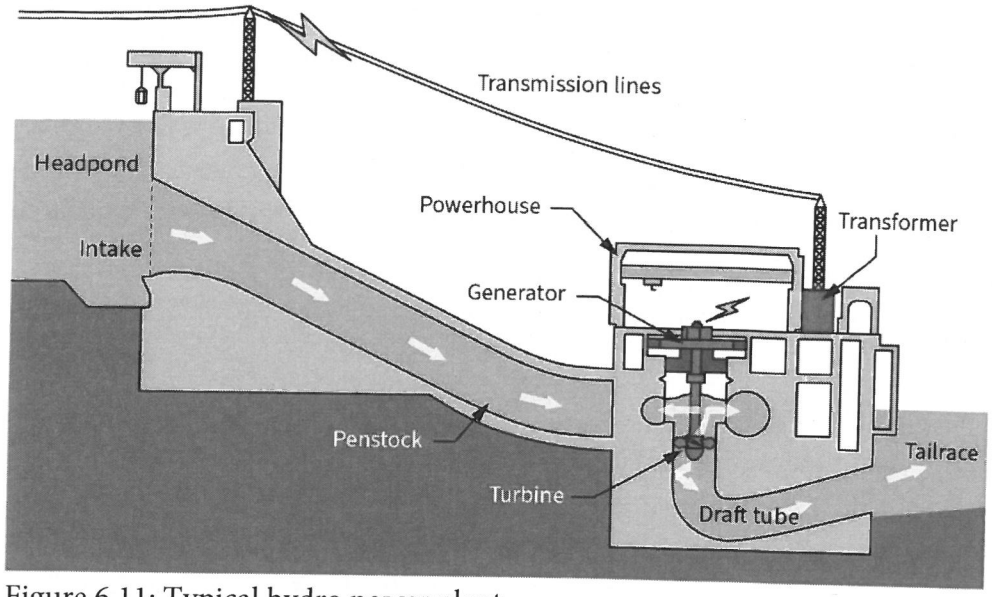


Figure 6.11: Typical hydro power plant

Types of hydroelectric systems

Note the different kind of hydroelectric systems in use.

Impoundment facilities

These systems utilize a dam to create a reservoir. Water is stored in this reservoir and released through turbines as needed. The potential energy of the water is converted into kinetic energy as it flows down, turning the turbines connected to generators that produce electricity. This type of system is the most common and allows for significant control over electricity generation, adapting to demand fluctuations.

Diversion facilities

Instead of using a dam, diversion facilities channel river water through canals to turbines. This method relies on the natural flow of the river and does not create a large reservoir. It is less intrusive to the local environment but may not provide the same level of electricity generation control as impoundment systems.

Pumped-storage facilities

These systems act like a battery by storing energy. During periods of low electricity demand, excess energy is used to pump water from a lower reservoir to an upper one. When demand increases, the stored water is released back down to generate electricity. This method is particularly useful for balancing supply and demand in the energy grid.

Advantages

- Renewable and clean: Hydroelectric systems produce electricity without burning fossil fuels, resulting in minimal pollution and greenhouse gas emissions.
- Reliable and flexible: They can quickly adjust to changes in electricity demand, making them a stable energy source compared to intermittent sources like wind and solar.
- Long lifespan: Hydroelectric plants can operate for many decades, often exceeding 50 years with relatively low maintenance costs once built.

Disadvantages

- Environmental impact: The construction of dams can disrupt local ecosystems, displace communities, and affect wildlife habitats. Reservoirs may also lead to the flooding of large areas of land.
- Dependence on water supply: Hydroelectric generation is reliant on adequate water flow, which can be affected by seasonal changes and climate variability, such as droughts.

6.2.2 Calculations on hydroelectric plants

These are the kinds of calculations involved for performing calculations on hydroelectric plants.

Gross head (H_g)

It is the height difference between the water level of the reservoir and the level of the tail water leaving the turbine. This head is used to determine the maximum amount of power that can be produced by the water.

Effective head

It is equal to the gross head minus the sum of all the head losses in the pipeline. This head is used to determine the power developed in the runner.

$H_e = H_g - h_f$

Where

h_f is losses in the penstock (m)

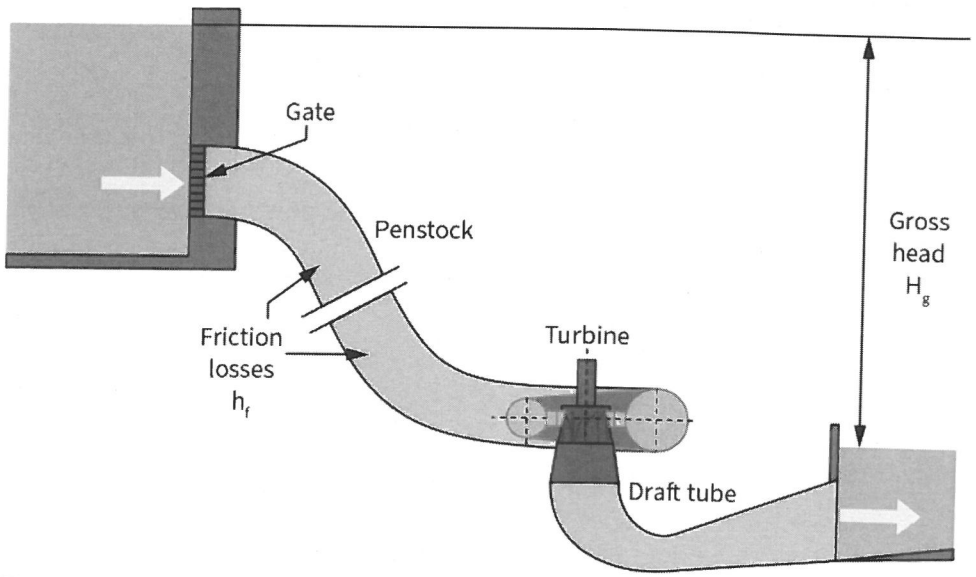


Figure 6.12: Gross head and friction losses

Water power (P_w): The power available of the stored water.

$$P_w = \rho g H_g Q$$

Runner power (P_r): The power developed by the runner.

$$P_r = \rho g H_e Q$$

Where

ρ is density of water ($1\ 000\ kg/m^3$)

Q is flow rate (m^3/s)

Shaft power: The output power available on the shaft

$$P_s = 2\pi NT$$

Where:

N is rotational frequency of shaft (rps)

T is torque on shaft (Nm)

Hydraulic efficiency

Hydraulic efficiency measures how effectively the turbine converts the hydraulic energy of the water into mechanical energy. It is the ratio between the power of the runner to the power of the water at inlet.

$$\eta_h = \frac{P_r}{P_w} = \frac{H_e}{H_g}$$

Mechanical efficiency

Mechanical efficiency assesses the losses in the mechanical components of the turbine, such as bearings on the shaft. It is the ratio between the power on the shaft to the power of the runner.

$$\eta_m = \frac{P_s}{P_r}$$

Overall efficiency

It is the ratio between the output power (P_s) and the input power (P_w). All the losses are considered. It is the product of hydraulic efficiency and mechanical efficiency.

$$\eta_o = \frac{P_s}{P_w} = \eta_h \times \eta_m$$

Example 6.8

A water turbine with an overall efficiency of 80% and a hydraulic efficiency of 90% works under a gross head of 70 m. The shaft has a rotational frequency of 600 rpm and power of 370 kW.

Calculate the following:

1. The flow rate
2. The effective head
3. The power developed in the runner
4. The torque on the shaft

Solutions

1.

$$P_w = \frac{P_s}{\eta_o} = \frac{370 \times 10^3}{0{,}8} = 462{,}5 \text{ kW}$$

$$Q = \frac{P_w}{\rho g H_g} = \frac{462{,}5 \times 10^3}{10^3 \times 9{,}81 \times 70} = 0{,}674 \text{ m}^3/\text{s}$$

2.

$$H_e = H_g \times \eta_h = 70 \times 0{,}9 = 63 \text{ m}$$

3.

$$P_r = \rho g H_e Q = 10^3 \times 9{,}81 \times 63 \times 0{,}674 = 416{,}25 \text{ kW}$$

4.

$$T = \frac{P_s}{2\pi N} = \frac{370 \times 10^3}{2\pi \times 10} = 5{,}889 \text{ kNm}$$

Example 6.9

A water turbine with an effective head of 20 m. The turbine discharges 75 m^3/s. The hydraulic efficiency is 90% and the mechanical efficiency is 92%.

Calculate the following:

1. The runner power
2. The output power
3. The gross head
4. The head lost to friction in the penstock

Solutions

1.

$$P_r = \rho g H_e Q = 10^3 \times 9{,}81 \times 20 \times 75 = 14{,}715 \text{ MW}$$

2.

$$P_s = P_r \times \eta_m = 14{,}715 \times 0{,}92 = 13{,}538 \text{ MW}$$

3.

$$H_g = \frac{H_e}{\eta_h} = \frac{20}{0{,}9} = 22{,}222 \text{ m}$$

4.

$$h_f = H_g - H_e = 22{,}222 - 20 = 2{,}222 \text{ m}$$

Example 6.10

A water turbine works under a gross head of 120 m. There is a 10% loss of head in the penstock due to friction due to a flow rate of 50 m^3/s. The turbine needs to produce 600 kNm of torque on the output shaft at a rotational frequency of 720 rpm.

Calculate the following:

1. The head loss due to friction
2. The power developed by the runner
3. The mechanical efficiency
4. The overall efficiency

Solutions

1.

$$h_f = 0{,}1 \times H_g = 0{,}1 \times 120 = 12 \text{ m}$$

2.

$$H_e = H_g - h_f = 120 - 12 = 108 \text{ m}$$

$$P_r = \rho g H_e Q = 10^3 \times 9{,}81 \times 108 \times 50 = 52{,}974 \text{ MW}$$

3.

$$P_s = 2\pi NT = 2\pi \times 12 \times 600 \times 10^3 = 45{,}239 \text{ MW}$$

$$\eta_m = \frac{P_s}{P_r} = \frac{45{,}239}{52{,}974} = 0{,}854 = 85{,}398\%$$

4.

$$P_w = \rho g H_g Q = 10^3 \times 9{,}81 \times 120 \times 50 = 58{,}86 \text{ MW}$$

$$\eta_o = \frac{P_s}{P_w} = \frac{45{,}239}{58{,}86} = 0{,}769 = 76{,}859\%$$

Exercise 6.2

1. A water turbine with an overall efficiency of 85% and a hydraulic efficiency of 90% works under a gross head of 150 m. The shaft transmits 800 kW of power with a torque of 12 kNm.

Calculate the following:

1.1 The flow rate

1.2 The effective head

1.3 The power developed in the runner

1.4 The speed of the shaft

2. A water turbine with an effective head of 320 m. The turbine has a discharge of 90 m^3/s. The hydraulic efficiency is 93% and the mechanical efficiency is 88%.

Calculate the following:

2.1 The runner power

2.2 The output power

2.3 The gross head

2.4 The head lost to friction in the penstock

3. A water turbine works under a gross head of 400 m. There is a 12% loss of head in the penstock due to friction due to a flow rate of 60 m^3/s. The turbine needs to produce 950 kNm of torque on the output shaft at a rotational frequency of 1 680 rpm.

Calculate the following:

3.1 The head loss due to friction

3.2 The power developed by the runner

3.3 The mechanical efficiency

3.4 The overall efficiency

Module summary

Wind speed – Wind speed is a critical factor in wind energy generation. The power available from wind is proportional to the cube of the wind speed.

Types of wind turbines

- Horizontal axis-Propeller type
- Vertical axis-Darrieus or Savonius

Infrastructure – The wind turbine infrastructure arrangement refers to the various components and systems that make up a wind farm or wind power plant.

Key factors to consider are

- Turbine spacing
- Foundations and layout
- Access roads and maintenance
- Orientation
- Grid connection
- Setback distances

Components

- Foundation – Provides stability
- Tower – Elevates the rotor blades
- Rotor blades – Converts wind energy into mechanical energy
- Hub – Secures blades and houses pitch control systems
- Nacelle – Houses gearbox, generator and wind control systems
- Gearbox – Speeds up rotor shaft revolutions
- Generator – Converts mechanical energy into electrical energy
- Yaw system - Orients the turbine according to wind direction
- Transformer - Steps up voltage of the electricity generated

Advantages of wind energy

- Wind energy is free.
- Very clean, non-polluting form of energy
- Renewable resource.
- No complex maintenance necessary
- Low operation cost.

Disadvantages of wind energy

- A higher initial investment.
- Cost of the machinery is high.
- Visual landscape harm
- High noise produced by the rotor blades
- Unsteady wind

Calculations

Wind speed

$$\frac{v}{v_o} = \left(\frac{H}{H_o}\right)^{\alpha}$$

Wind power

$$P_w = \frac{\dot{m}v^2}{2}$$

$$P_w = \frac{\rho A v^3}{2}$$

Power coefficient

$$C_p = \frac{P_r}{P_w}$$

Mechanical efficiency

$$\eta_m = \frac{P_g}{P_r}$$

Electrical efficiency

$$\eta_e = \frac{P_o}{P_g}$$

Overall efficiency

$$\eta_o = \frac{P_o}{P_w}$$

$$\eta_o = \eta_m \times \eta_e \times C_p$$

Optimal tip speed ratio

$$\lambda = \frac{v_t}{v} = \frac{4\pi}{n}$$

Betz power coefficient

$$C_{pm} = \frac{16}{27} = 0{,}593$$

eLinks

Visit these links to learn more about wind turbines:

- youtu.be/LNXTm7aHvWc
- youtu.be/xws3Hi3mujs

Summative assessment

1. The rotor power of a wind turbine is 1 MW at a certain wind speed.

Calculate the following:

1.1 The power if the air density increases by 10% (2)

1.2 The power if the blade length is decreased by 25% (2)

1.3 The power if the wind speed increases by 50% (2)

2. The wind has a speed of 5 m/s at a height of 10 m over an area with many trees, resulting in a friction coefficient of 0,25. The hub of the horizontal axis turbine used is 160 m above the ground. The density

of the air at this height is 1,18 kg/m^3. The rotor has a length of 32 m. The power coefficient is 70% of the Betz coefficient. The overall efficiency is 30%.

Calculate the following:

2.1 The power of the wind (5)

2.2 The rotor power (2)

2.3 The output power (2)

3. The rotor of a horizontal axis wind turbine has a length of 28 m. The rotor has 3 blades. The wind speed is 8 m/s. Assume the density as 1,3 kg/m^3 and the power coefficient as 0,4.

Calculate the following:

3.1 The optimum rotational speed of the rotor (3)

3.2 The rotor power (2)

3.3 The maximum rotor power possible (2

4. The rotor blade of a horizontal axis wind turbine has a length of 32 m. The wind speed is 12 m/s. Assume C_p as 0,35 and the density as 1,3 kg/m^3.

Calculate the following:

4.1 The rotor power (2)

4.2 The rotor power for a blade length increase of 30% (2)

4.3 The percentage increase in power (2)

5. The rotor blade of a horizontal axis wind turbine has a length of 36 m. The wind speed is 6 m/s. Assume C_p as 0,3 and the density as 1,23 kg/m^3.

Calculate the following:

8.1 The flow rate (3)

8.2 The effective head (2)

8.3 The power developed in the runner (2)

8.4 The torque of the shaft (2)

9. A water turbine with an effective head of 220 m. The turbine has a discharge of 60 m^3/s. The hydraulic efficiency is 93% and the mechanical efficiency is 90%.

Calculate the following:

9.1 The runner power (2)

9.2 The output power (2)

9.3 The gross head (2)

9.4 The head lost to friction in the penstock (2)

10. A water turbine works under a gross head of 250 m. There is a 7% loss of head in the penstock due to friction due to a flow rate of 40 m^3/s. The turbine needs to produce 550 kNm of torque on the output shaft at a rotational
frequency of 1 350 rpm.

Calculate the following:

10.1 The head loss due to friction (2)

10.2 The power developed by the runner (2)

10.3 The mechanical efficiency (2)

10.4 The overall efficiency (2)

Total: 100 marks

Calculate the following:

5.1 The rotor power (2)

5.2 The rotor power if wind speed decreases by 30% (2)

5.3 The percentage decrease in power (2)

6. The rotor blade of a horizontal axis wind turbine has a length of 24 m. The wind speed is 8 m/s. Assume the density as 1,2 kg/m^3. The output power of the turbine is 350 kW. Overall efficiency is 25% and electrical efficiency is 92%. The power coefficient is 50% of the mechanical efficiency.

Calculate the following:

6.1 The wind power (2)

6.2 The electrical power (2)

6.3 The rotor power (3)

7. The rotor blade of a horizontal axis wind turbine has a length of 50 m. The wind speed is 8 m/s. The rotor rotates at 12 rpm. Assume C_P as 0,4 and the density as 1,225 kg/m^3. The gearbox has a ratio of 1:30 and an efficiency of 87%. The overall efficiency is 45%.

Calculate the following:

7.1 The mass rate of the air (2)

7.2 The rotor power (2)

7.3 The torque on the high speed shaft (2)

7.4 The energy output per day (3)

8. A water turbine with an overall efficiency of 84% and a hydraulic efficiency of 94% works under a gross head of 130 m. The shaft transmits 750 kW of power with a torque of 600 rpm.

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